Housing and the Labor Market: Time to Move and Aggregate Unemployment*

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May 25, 2009

Abstract

The Mortensen-Pissarides model with unemployment benefits and taxes has been able to account for the variation in unemployment rates across countries but does not explain why geographical mobility is very low in some countries (on average, three times lower in Europe than in the U.S.). We build a model in which both unemployment and mobility rates are endogenous. Our findings indicate that an increase in unemployment benefits and in taxes does not generate a strong decline in mobility and accounts for only half to two-thirds of the difference in unemployment from the US to Europe. We find that with higher commuting costs the effect of housing frictions plays a large role and can generate a substantial decline in mobility. We show that such frictions can account for the differences in unemployment and mobility between the US and Europe.

*We thank Nobu Kiyotaki and seminar participants at Rogerson-Shimer-Wright NBER group meeting at the Atlanta Fed, INSEE-Crest, the ADRES-EDHEC conference in Paris (Labour Markets Outcomes, A Transatlantic Perspective), the Transatlantic conference IZA-SOLE in Bonn, Queens University, University of Toronto, University of Iowa, Warwick University, the first annual matching conference in Malaga, Spain, Northwestern University, UC Davis, UC Irvine.

1 Introduction

The Mortensen-Pissarides model has been shown to successfully explain cross-country differences in unemployment and unemployment spells with two labor market institutions: unemployment benefits and taxes. Mortensen and Pissarides (1999) highlights the fact that roughly half of the mileage between a US unemployment rate (6%) to a European one (11%) can be explained by each institution. However, there are other differences in labor markets between countries left unaccounted for, such as geographical mobility. The residential mobility rate (both inter and intra-state) is three times higher in the US than the corresponding rates in Europe, and this seems to be a major source of non-resilience of unemployment in Continental Europe.

To study this specific difference, we build a modified version of Mortensen-Pissarides in order to capture geographical mobility. Workers receive offers characterized by a commute distance, and have the possibility to move conditional on receiving housing offers. Within a very parsimonious setup, our model captures job acceptance decisions, decisions to move to another dwelling and job creation decisions, with both search and matching frictions in the housing and labor market. The model also captures the complementarity between commuting and moving decisions.

In our model, a job location has an associated commuting time that may affect the job acceptance decision. Obstacles to mobility, such as regulations in the housing market, will affect the reservation strategy of workers. Thus, aggregate unemployment will have an effect on the functioning of the housing market. The model can be thought of as the "dual" of typical search models of the labor market. In particular, in the standard search setup there may be a non-degenerate distribution of wages but distance is degenerate. In our model, the distribution of wages is degenerate but there exists instead a non-degenerate distribution of distance from one's job. Gaumont, Schindler, and Wright (2006) provide an example of how a non-degenerate wage distribution can arise from ex-ante homogeneous agents. In their model, when a worker chooses a job they also randomly choose a "cost to taking the job" that can be interpreted in the context of our model as a commuting cost.

We show that there is a complementarity between institutional ingredients that make commute

more expensive (gasoline tax, price of cars and car insurance) and frictions in the housing market. Since both components are higher in Europe than in the US, we have a potentially attractive explanation of European unemployment; however, we also know from earlier work, notably from Mortensen and Pissarides (1999) that there is also a complementarity between various "pure" labor market institutions, such as unemployment benefits and payroll taxes.

Therefore, we provide a quantitative exercise to capture the effect of changes in those four factors (benefits, labor taxes, commute costs and housing frictions) on unemployment, unemployment duration and household's mobility. Our findings indicate that an increase in unemployment benefits and in taxes do not generate a strong decline in mobility and account for only two-thirds of the required increase in unemployment from the US to Europe. We find that with higher commuting costs, like those in Europe, the effect of housing frictions play a large role and can generate a decline in mobility similar to that in Europe, roughly one-third of that in the U.S.

In a related model, Head and Lloyd-Ellis (2008) use a spatial model of housing and show that home owners are less mobile than renters, but that the effect of home ownership on unemployment is quantitatively small. In our paper there is no distinction between owning a home or renting and therefore we abstract from such differences. We view our paper as complentary to theirs as they focus mainly on frictions between housing markets.

Section 2 provides a comparison of mobility and commuting costs for the US and EU. Section 3 presents the model with labor market and housing frictions.Section 4 describes the optimal strategies and equilibrium. Section 5 describes how frictions in the housing market affect mobility and unemployment rates. Section 6 lays out the calibration strategy and parameters. Section 7 concludes.

2 Empirical motivation

This paper starts with four facts. First, Table 1 reveals that 15.5% of American residents move yearly for one reason or another. In comparison, data for fifteen European Union countries show

that less than 5% of residents move yearly.[1]

Table 1: Mobility in the U.S. and E.U.

	US	**EU15**
Mobility rate	15.5%	4.95%
Share within county / area	0.58	0.83
Share between county / area	0.42	0.17

Second, Table 2 provides an overview of reasons for moving for those that moved. In the U.S., most of the intra-county mobility is house related (65.4%), and only a small fraction (5.6%) move within a county for job related reasons. In contrast, work related mobility is approximately 33% of inter-county moves. Family related mobility is a fairly constant fraction of all moves. In the E.U., perhaps surprisingly, there is a very similar pattern. Although residents in the U.S. move about three times the rate of those in the E.U., the reasons why they move are roughly similar. This is our second fact. This observation suggests that the shocks that affect mobility are similar across the countries; yet, there is substantially less overall mobility in the E.U. [2]

Third, there are large geographical mobility differences between countries. Scandinavian countries report mobility rates which are close to the US rates, while Southern European countries exhibit the lowest of all rates. Part of the differences may be due to socio-cultural factors, such as family attachment, or to differences in regions leading to higher mobility costs. But objective factors can be identified too: for example rental housing market regulations as shown by Djankov,

[1]Data for the U.S. comes from Geographic Mobility, Current Population Reports, P20-538, U.S. Census, May, 2001. Data for the E.U. comes from the European Community Household Panel 1999-2001. Mobility data for the U.S. come from the U.S. Census 2000. The two data sources cover essentially the same time period and ask very similar questions, making for a useful comparison of mobility patterns across countries. Although the paper does not address short vs. long distance moves (this is addressed in a companion paper Rupert, Stancanelli, and Wasmer (2009)), we include those data for completeness. About 58% of the moves are within county. The share of moves within an area/locality is higher, although at this level of disaggregation it is difficult to strictly compare to U.S. counties.

[2]Why People Move: Exploring the March 2000 Current Population Survey, Current Population Reports, P23-204, May, 2001, U.S. Department of the Census. The Census questions are actually more precise about the exact reasons for mobility, while no detailed questions are available in the ECHP. Table 9 in Appendix gives the details for the U.S.

Table 2: Reasons for Moving, US and EU15

	Proportions, US				Proportions, EU15		
All pop. (1+)	intra-county	inter-county	all		intra-area	inter-area	all
Work related	5.6%	31.1%	16.2%	Job related	7.61%	40.0%	14.3%
Family related	25.9%	26.9%	26.3%	Personal Reason	31.6%	29.8%	31.3%
House related	65.4%	31.9%	51.6%	House Related	59.1%	28.1%	52.7%
Others	3.0%	10.1%	6.0%	Not Available	1.7%	2.11%	1.8%
All reasons	100%	100%		All reasons	100%	100%	100%

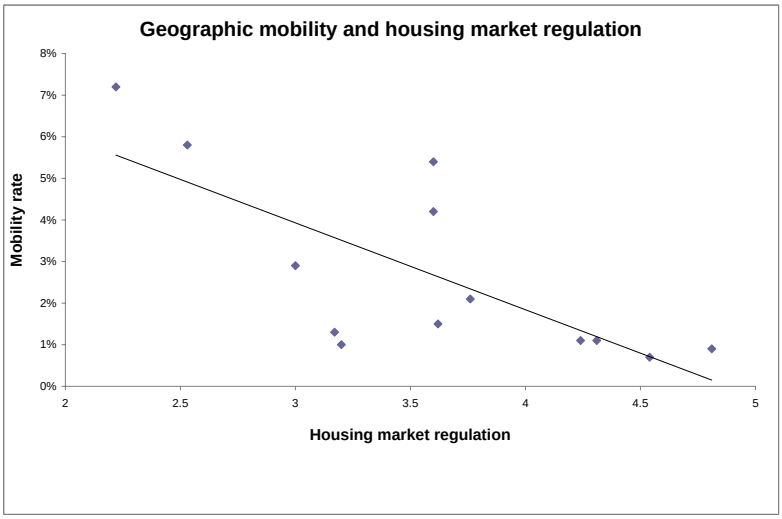
La Porta, Lopez de Silanes, and Shleifer (2003). This is a composite index based on the difficulty to evict a tenant, reflecting the complexity and the length of the procedure at various stages (pre-trial, process of trial, execution of the court decision). In Table 8, in the appendix, we report the numbers for 13 European countries and contrast the indices with mobility rates and with average unemployment rates. Figure 1 plots the data in Table 8 and displays a strong, negative correlation between housing market regulations and mobility rates across European countries. This is our third fact.

Fourth, there are large differences between countries in the consumer price of gas. Table 3 shows that the price of gas in many European countries is two or three times that in the United States. This in turn implies that commute costs are going to vary quite significantly. A good example of this cost differences is accounted for by tax deductions allowed by the Internal Revenue Service in the US and by the Tax administration in France; for the IRS, the standard mileage rate in 2007 for the use of a car is 0.485$ per mile, that is 0.20 euro per kilometer. For the Tax authority in France, for less than 5000 km, this is 0.514 euro per kilometer for a median size car.[3]

We then use these four facts to develop a model that allows for interaction between the labor market and the housing market, both being markets with *frictions*, to address three questions: Can the cross-country differences in mobility be accounted for by a frictional parameter in the housing

[3]Bulletin Officiel des Impôts, January 2007. The rate is actually progressive with power, from 0.37 euro / km for smaller cars to 0.67 euro per km for more expensive cars.

Figure 1: Housing Regulations and Mobility



market as fact 3 suggests? Is the level of unemployment in a country affected by imperfections in the housing market; and if yes, through which mechanisms? We ask this question because of the large difference in observed unemployment rates. In July of 2007 the unemployment rate in the United States was 4.6 % while in the four large continental countries the unemployment rates were substantially higher: The Spanish unemployment rate was over 8%, Italy around 7%, Germany over 8% and France above 9%. Finally, quantitatively, would a reform of the housing market lead to a dramatic improvement in labor market performance in the US or in Europe?

3 Model

3.1 Preferences and Search in the Housing Sector

Time is continuous and individuals discount the future at rate r. Individuals live in dwellings, defined as a bundle of services generating utility to individuals or a household. The defining

Table 3: Gasoline Prices

	US$/gal.
United States	1.80
France	5.60
Belgium	6.17
Germany	5.79
Italy	5.79
Hungary	7.51
Netherlands	6.47
Norway	6.09

Source: German Technical Corporation, 2007

characteristic of a dwelling, however, is that the services it provides are attached to a fixed location. The services can, of course, depend on the quality of the dwelling and its particular location. Amenities such as space, comfort, proximity to theaters, recreation, shops and the proximity to one's job increase the utility of a given dwelling. The dwelling may also be a factor of production of home-produced goods. In addition, the dwelling could be a capital asset. For these services, individuals pay a rent or a mortgage.

In this paper we focus on one particular amenity, distance to work. Because a dwelling is fixed to a location, the commuting distance to one's job, ρ, becomes an important determinant of both job and housing choice. We assume that space is symmetric, in the sense that the unemployed have the same chance of finding a job wherever their current residence, implying that ρ is a sufficient statistic determining both housing and job choice. We call this property the isotropy of space: wherever an individual is, space looks the same.

Agents face two types of housing shocks. First, they may receive a family shock that arrives according to a Poisson process with parameter δ. The shock changes the valuation of the current location, necessitating a move. This shock can be thought of as a marriage, divorce, the arrival of children, deterioration of the neighborhood, and so on. Upon the arrival of the shock they make one draw from the existing stock of housing vacancies, distributed as $G_S(\rho)$.[4] Note that agents

[4]The one draw assumption is not very strong. It is equivalent to making up to N independent draws, in which case

may sample from the existing stock of houses at any time.

Second, agents randomly receive new housing opportunities that (possibly) allows them to relocate closer to their job. These arrivals are assumed to be Poisson with parameter λ_H. The distribution of *new* vacancies is given as $G_N(\rho)$.

We make the simplifying assumption that the efficiency of the housing market can be captured in a single variable, λ_H, the parameter determining frictions in the housing market. An increase in λ_H means there are more arrivals of opportunities to find housing. As λ_H approaches infinity, housing frictions go to zero. The main idea behind λ_H is that agents may not move instantaneously to their preferred location. Such restrictions might arise from length of lease requirements or eviction policies. In the Appendix we discuss the relationship between housing market regulations and housing offers, λ_H, but for now we assume it represents housing market frictions. This implies that the rent or mortgage is such that utility across dwellings will be equalized to reflect any differences in amenities, a fact that results from the assumption that space (distance) is isotropic.

3.2 Labor Market

Individuals can be in one of two states: employed or unemployed. While employed, income consists of an exogenous wage, w.[5] There is no on-the-job search, yet a match may become unprofitable, leading to a separation, which occurs exogenously with Poisson arrival rate s.

Unemployed agents receive income b, where b can be thought of as unemployment insurance or the utility from not working. While unemployed, job offers arrive at Poisson rate p, indexed by a distance to work, ρ , drawn from the cumulative distribution function F_J.

Let $E(\rho)$ be the value of employment for an individual residing at distance ρ from the job. Let U be the value of unemployment, which does not depend on distance, given the symmetry assumption made above. We can now express the problem in terms of the following Bellman

it is like one single draw from a distribution $(G_S)^N$. See Lemma 1 in Quentin et al. (2006).

[5] An exogenous wage greatly simplifies the analysis because the wage does not depend on commute distance. However, in the calibration we allow the wage to depend on taxes and benefits and examine alternative choices for the parameters as a robustness check on the importance of this assumption. In the Appendix we show how the model could be recast in terms of a wage-posting framework.

equations:

$$
\begin{aligned}
(r+s)E(\rho) &= w-\tau\rho+sU+\lambda_H\int \max\left[0,(E(\rho')-E(\rho))\right]dG_N(\rho') \qquad (1)\\
&\quad +\delta\int \max[U-E(\rho),E(\rho'')-E(\rho)]dG_S(\rho'')\\
(r+p)U &= b+p\int\int \max[U,E(\rho'),E(\rho^{''})]dF_J(\rho')dG_S(\rho''), \qquad (2)
\end{aligned}
$$

where τ is the per unit cost of commuting and ρ is the distance of the commute.[6] Eq. 1 states that workers receive a utility flow $w-\tau\rho$; may lose their job and become unemployed – in which case they stay where they are; they receive a housing offer from the distribution of new vacancies G_N, which happens with intensity λ_H, in which case they have the option of moving closer to their job; and finally may receive a family shock δ, and need to relocate.

Eq. 2 states that the unemployed enjoy b; receive a job offer with Poisson intensity p, at a distance ρ', from the distribution $F_J(\rho)$. They have the option of rejecting the offer if the distance is too far, but also have the option to move instantaneously if they find a residence in the stock of existing vacant units. To the extent that ρ' and ρ'' are independent draws, this means that there is a distribution, F, combining F_J and G_S such that the integral terms can be rewritten as $\int \max[U,E(\rho)]dF(\rho)$, where ρ is the minimum of the two draws: $\rho = Min(\rho',\rho'')$.[7]

4 Optimal Behavior, Equilibrium and Steady-states

4.1 Reservation Strategies

We now derive the job acceptance and moving strategies of individuals. Observe that E is downward sloping in ρ, with slope

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r+s+\lambda_H P_W+\delta}, \qquad (3)$$

[6] It is possible to reinterpret commuting as any non-pecuniary aspect of the job.

[7] We prove in the appendix that $1-F(\rho) = (1-F_J(\rho))(1-G_S(\rho))$.

where P_W is the probability of moving conditional on receiving a housing offer. Note that $0 < P_W < 1$ and possibly depends on ρ. The function $E(\rho)$ is monotonic so that there exists a well-defined reservation strategy for the employed, with a reservation distance denoted by $\rho^E(\rho)$, below which a *housing offer* is accepted. Note that there is state-dependence in the reservation strategy of the employed, $\rho^E(\rho)$, with presumably $d\rho^E(\rho)/d\rho > 0$. Evidently, the further away the tenants live from their job, the less likely they will be to reject a housing offer.

After some intermediate steps (described in Appendix), we can show that the slope of $E(\rho)$ is given by

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r+s+\lambda_H G_N(\rho)+\delta}. \tag{4}$$

Next, in the absence of relocation costs (this case is studied in the Appendix), tenants move as soon as they get a dwelling offer closer to their current one, implying

$$\rho^E(\rho) = \rho.$$

Denote by ρ^U the reservation distance for the unemployed, below which any *job offer* is accepted, it is defined by

$$E(\rho^U) = U.$$

Using the fact that $E(\rho^U) = U$,

$$\begin{aligned} b + p\int_0^{\rho^u} [E(\rho') - U]dF(\rho') \\ = w - \tau\rho^U + \lambda_H \int_0^{\rho^U} [E(\rho') - U]dG_N(\rho') + \delta \int_0^{\rho^u} [E(\rho'') - U]dG_S(\rho''). \end{aligned} \tag{5}$$

Integrating Eq. 5 by parts gives:

$$\rho^U = \frac{w-b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) - pF(\rho)}{r+s+\lambda_H G_N(\rho)+\delta} d\rho. \tag{6}$$

The determination of ρ^U is shown in Figure 2.

Note that a job separation can occur in two ways. First, as mentioned above, there is an exoge-

nous shock, s, inducing a separation from the match. Second, workers may receive a family shock, δ, requiring them to redraw from the vacant housing stock distribution, G_S, but are unable to find a sufficiently close dwelling to the current job and (optimally) quit. That is, job separations are given by

$$\sigma = s + \delta(1 - G_S(\rho^u)). \tag{7}$$

With this specification the model is quite parsimonious, since a single variable, ρ, determines several dimensions of choice:

1. *Job acceptance*: $F(\rho^U)$;
2. *Residential mobility rate*: $\int \lambda_H G_N(\rho)$ over the distribution of commute distance of employed workers $d\Phi$;
3. *Quit rate due to a family shock*: $\delta(1 - G_S(\rho^U))$.

4.2 Free Entry

Assuming free entry of firms, and defining $\theta = \frac{V}{U}$ as labor market tightness, we have

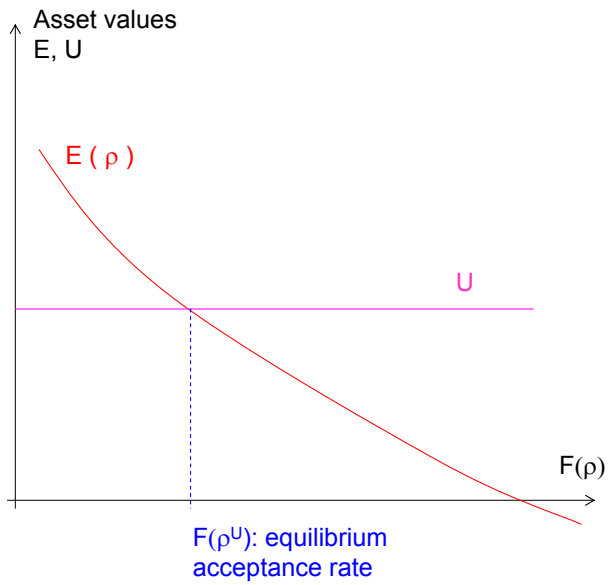
$$\frac{y - w}{r + \sigma} = \frac{c}{q(\theta)P_F},$$

where P_F is the rate of acceptance of job offers by the unemployed, as expected from the viewpoint of the firm. We assume, still by symmetry, that the distribution of contacts between the firm and unemployed workers is given as $F(\rho)$, so that $P_F = F(\rho^U)$. This generates a positive link between θ and ρ^U since $q'(\theta) < 0$, characterized by:

$$q(\theta)F(\rho^U) = \frac{c(r + \sigma)}{y - w}. \tag{8}$$

The intuition is quite simple. The firm's iso-profit curve at the entry stage depends negatively on both θ (as a higher θ implies more competition between the firm and the worker) and on ρ^U (as

Figure 2: Determination of ρ_u



more of their offers will be rejected because of distance). The zero-profit condition thus implies a positive link between θ and ρ^U. Note that this relation is independent of λ_H. On the other hand, ρ^U is determined through (Eq. 6). It is decreasing in $p(\theta)$ and thus in θ, as can be seen in (Eq. 21). When there are more job offers (higher θ) workers can wait for offers closer to their current residential location; they are pickier. The two curves are represented in (ρ^U, θ) space in Figure 3.

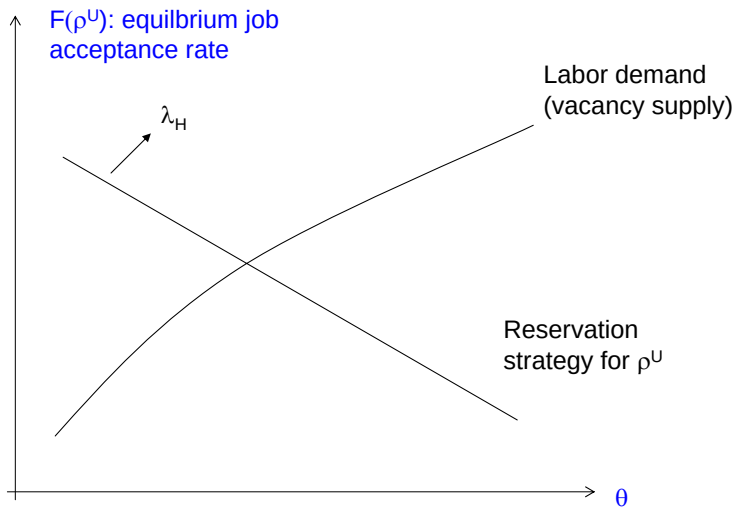
4.3 Unemployment and the Beveridge Curve

Recapitulating, an increase in λ_H, the efficiency of the housing sector, raises the acceptance rate of job offers, increasing θ and thus increasing job offers by firms.

Letting $p(\theta) = \theta q(\theta)$, the steady state unemployment rate is given as

$$u = \frac{\sigma}{\sigma + p(\theta)F(\rho^U)}. \tag{9}$$

Figure 3: Vacancies and Reservation Strategy



In terms of a Beveridge Curve representation (vacancy and unemployment space), increasing λ_H shifts the Beveridge curve inward (less structural mismatch) and also leads to a counter-clockwise rotation of θ. A graphical representation of this result is shown in Figure 4.

5 Housing Frictions and Mobility

5.1 The Effect of Regulations in the Housing Market

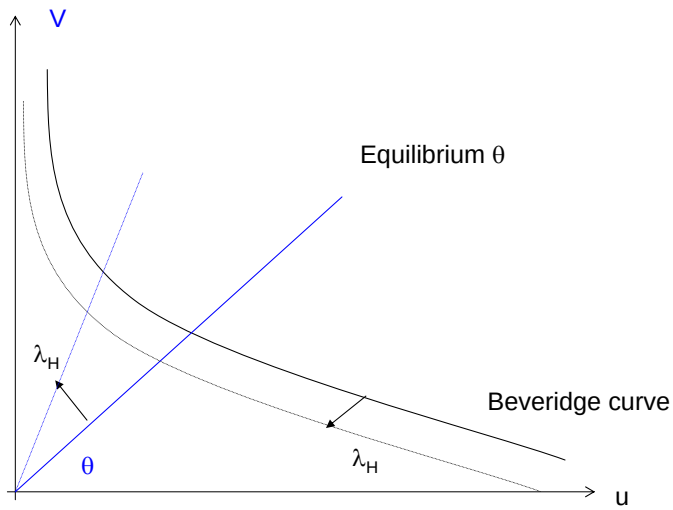
It is now possible to determine how housing frictions affect the decisions of workers and firms.

Proposition 1 *An increase in λ_H makes the unemployed less choosy about jobs: $\partial\rho^U/\partial\lambda_H > 0$.*

Proof. See Appendix. ■

Next, differentiating (Eq. 8) and using Proposition 1, we can determine the effect of housing frictions on job creation:

Figure 4: Beveridge Curve



Proposition 2 *An increase in λ_H increases job creation: $\partial\theta/\partial\lambda_H > 0$.*

Proof. Same as Proposition 1. ■

This is an indirect effect caused by more job creation through the higher job acceptance rate of workers. Another interpretation of this effect is that *firms don't like to create jobs where workers have no place to live.*

Using these results it is now possible to determine the effect of housing market frictions on unemployment.

Proposition 3 *An increase in λ_H has three effects on unemployment:*

- *it reduces the quit rate in case of a δ-shock;*
- *it raises the job acceptance rate of workers (through a higher thresehold ρ^U);*
- *it raises θ (Proposition 2) and thus job creation.*

Proof. See Appendix. ■

5.2 Distribution of commute distance

Let $\Phi(\rho)$ be the steady-state distribution of employed workers living at a location closer than ρ. Φ is governed by the following law of motion, for all $\rho < \rho^U$:

$$\begin{aligned}(1-u)\frac{\partial \Phi(\rho)}{\partial t} =& upF(\rho) + (1-u)(1-\Phi(\rho))\left\{\lambda_H G_N(\rho) + \delta G_S(\rho)\right\} \\ & - \delta(1-u)\Phi(\rho)(1-G_S(\rho)) - (1-u)\Phi(\rho)s\end{aligned} \tag{10}$$

Eq. 10 states that the number of people residing in a location at a distance less than ρ from their job changes (either positively or negatively) due to:

- (+) the unemployed, u, receiving a job offer at rate p with a distance closer to ρ with probability $F(\rho)$;
- (+) the employed, $1-u$, who are further away from the current distance ρ (a fraction $1-\Phi(\rho)$), who receive an offer in the housing market with intensity λ_H closer to ρ with probability $G_N(\rho)$;
- (+) the employed, $1-u$, who are further away from the current distance ρ (a fraction $1-\Phi(\rho)$), who face a δ-shock that brings them closer to ρ after sampling in the stock G_S;
- (-) the employed, $1-u$, who were at a distance less than ρ (a fraction $\Phi(\rho)$), receive a δ-shock that brings them further away from ρ after sampling in the stock G_S; note that a fraction of them would even quit if their new ρ is above ρ^U;
- (-) the employed, $1-u$, who receive an s-shock, that is, exogenous job destruction.

In steady state and for all $\rho < \rho^u M$:

$$\Phi(\rho) = \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) + pF(\rho)\frac{u}{1-u}}{\lambda_H G_N(\rho) + \delta + s} \tag{11}$$

$$= \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) + \frac{F(\rho)}{F(\rho^u)}\sigma}{\lambda_H G_N(\rho) + \delta + s} \tag{12}$$

$$= \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) + \frac{F(\rho)}{F(\rho^u)}\sigma}{\lambda_H G_N(\rho) + \delta + \sigma} \leq 1 \tag{13}$$

The second line above is obtained by replacing u with its steady-state expression in (Eq. 9). Note that for $\rho = \rho^U$, $\Phi(\rho^U) = 1$ as no unemployed individual ever accepts a job offer farther away from a job than ρ^U.

5.3 Log Linearization

First, consider the special case: $\lambda_H \to \infty$. In the case where housing frictions go to zero, the model collapses to $\Phi(\rho) = 1$, meaning that all workers will be located epsilon-close to their job. The job acceptance decision is indeterminate since we now have

$$\rho^U = \frac{w-b}{\tau} + \int_0^{\rho^U} d\rho.$$

The intuition is straightforward: if $w > b$, all job offers are accepted, meaning that ρ^U goes to infinity. Therefore, we obtain the standard Pissarides value for tightness: $q(\theta^P) = \frac{c(r+\sigma)}{y-w}$ with $\theta^P > \theta^*$ where θ^* is equilibrium tightness in our mode. In addition,

$$\frac{q(\theta^P)}{q(\theta^*)} = F(\rho^U) < 1$$

and therefore, with $q(\theta + d\theta) = q(\theta) + q'(\theta)d\theta = q(\theta)(1 + \eta_q d\theta/\theta)$, we have

$$\frac{q(\theta^P)}{q(\theta^*)} = 1 + \eta_q d\theta/\theta^* = F(\rho^U)$$

hence

$$\frac{d\theta}{\theta^*} = \theta^P - \theta^* = \frac{1 - F(\rho^U)}{-\eta_q} > 0$$

The percentage change in tightness is of the order of magnitude of the rejection rate of job offers divided by the elasticity of matching. Since the percentage change in unemployment is the percentage change in tightness multiplied by $(1-u)\eta_p$, the overall change in unemployment due to imperfect housing markets is of the order of magnitude of the fraction of rejected offers $1-F(\rho^U)$ if $\eta_p \simeq -\eta_q \simeq 0.5$.

6 Calibration

6.1 Calibration targets

In this section we will match the model to the data, in particular the mobility rates. We therefore need to calculate the mobility rate from the model. Denote by M_K^S the number of movers of status $S=(U,E)$ (unemployed, employed) and for reason $K=(J,D)$ (job-related or family-related), we have:

1. Job-related mobility of the employed (those with a job but relocate once they sample a better housing location):

$$M_J^E = (1-u)\lambda_H \int_0^{\rho^U} G_N(\rho)d\Phi(\rho) \tag{14}$$

$$= (1-u)\lambda_H \left[G_N(\rho^U) - \int_0^{\rho^U} g_N(\rho)\Phi(\rho)d\rho \right], \tag{15}$$

   where the second line is found by integrating by parts and noticing that $\Phi(\rho^U)=1$.

2. Job-related mobility of the unemployed (those who have a job offer, accept it with probability $G(\rho^U)$ and may relocate if they drew a location from G_S closer from their current ρ):

$$M_J^U = up \int_0^{\rho^U} G_S(\rho)dF_J(\rho)$$

3. Family-related mobility:

$$
\begin{aligned}
M_D^U &= u\delta \\
M_D^E &= (1-u)\delta \\
M_D^{E+U} &= \delta
\end{aligned}
$$

Note that in M_D^E, some workers quit their job (a fraction $1-G_S(\rho^U)$) since they did not find acceptable housing in the current stock.

6.2 Taxes, Benefits and Wages

So far, the model has abstracted from taxes. As shown in Mortensen and Pissarides (1999) and Prescott (2004), taxes and benefits can explain much, if not all, of the variation in unemployment rates across countries. We therefore introduce a tax on labor denoted by t which will be set to 0.22 for the US and 0.4 for Europe. However, it is quite well known that taxes on labor lower wages and therefore that there is a "crowding out" effect: A one percentage point increase in taxes does not necessarily imply a one percentage point increase in labor costs. The net effect depends, in principle, on the elasticity of demand, supply and the bargaining power of workers. In the model developed so far, it is possible to make wages endogenous and introduce a bargaining game. However, the cost is to lose most of the simplicity of the model as the wage will then depend on commute distance. We take an alternative route here: Keep an exogenous wage, but argue that part of the effect of taxes is diluted due to a crowding out parameter denoted by ε.[8] In short, if taxes are t, the total labor cost is denoted by $w(1+\varepsilon t)$ and the net wage of workers is $w[1-(1-\varepsilon)t]$. It follows that the main equations of the model become:

$$
q(\theta)F(\rho^U) = \frac{c(r+\sigma)}{y-w(1+\varepsilon t)} \tag{16}
$$

$$
\rho^U = \frac{w[1-(1-\varepsilon)t]-b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho)+\delta G_S(\rho)-pF(\rho)}{r+s+\lambda_H G_N(\rho)+\delta} d\rho, \tag{17}
$$

[8] In the Appendix we derive results for endogenous wages in a wage posting model

while the stock-flow equations and the rate of unemployment are unchanged. We set ε to be 0.35, implying that a 10% increase in labor taxes generates a 3.5% increase in labor costs and a 6.5% decrease in the net wage of workers.[9]

Finally, it is unrealistic to assume that a change in unemployment benefits has no direct effect on wages, and only an indirect effect on the average wage in the economy through an increase in reservation wages. This is why in the calibration we allow for the direct effect by arguing that $w(b) = w_{US} + (1-\beta)(b - b_{US})$ where any additional dollar of unemployment compensation raises the wage by 1-β where β can be thought of as the bargaining power of workers: This is the same specification as that emerging from Nash-bargaining. We set $\beta = 0.5$ so that the bargaining power is symmetric. We set $w_{US} = 0.6$ and the output generated in the match is normalized to $y = 1$. Labor taxes in the US are given by $t = 0.22$ and unemployment benefits are $b = 0.25$. Labor taxes in Europe are $t = 0.4$ and unemployment benefits are equal to $b = 0.4$ (for a wage of 0.627). So, roughly speaking a replacement rate of 42% in the US and 64% in Europe.

One could, of course, fully endogenize wages (e.g. Nash-bargaining) but then would face the same problem as discussed above (the wage would depend on distance) and hence we choose to preserve the simplicity of the model and a realistic, but ad hoc, wage in the calibration.

6.3 Calibration

The time period is one month and the interest rate, r, is set to 0.0033, corresponding to an annual rate of 0.04. We calibrate to the mobility rate of the employed, 15.5% annually between March 1999 and March 2000, so the target is (15.5/12)%. The number for the employed that move comes from the Bureau of the Census.[10] Of the roughly 31 million persons who moved during that year, 22.3 million of them were employed, 1.5 million unemployed and 7.8 million out of the labor force.

[9] Assuming that ε is being approximated by $\varepsilon^{LS}/(\varepsilon^{LS} + \varepsilon^{LD})$ where ε^{LS} and ε^{LD} are the absolute elasticities of labor supply and labor demand, this would imply that $\varepsilon^{LD}/\varepsilon^{LS} = 2$.

[10] Why People Move: Exploring the March 2000 Current Population Survey, P23-204, Bureau of the Census, May, 2001.

We have three distributions to account for: G_N, new housing offers, G_S, the stock of houses and F, job offers. We assume that these distributions are represented by exponential functions with parameter α: $F = G_N = 1 - e^{-\alpha\rho}$ and $G_S = 1 - e^{-(\alpha/3)\rho}$.

To calculate α and τ, we proceed as follows. First, Table 4 shows the distribution of commute times times from the Census 2000 as a fraction of total hours worked. The median commuter spends 0.083 of its working time to commute.

Table 4: Commute time as a fraction of total hours worked

	US	Fr
Mean	0.102	0.079
10th percentile	0.020	0.021
25th percentile	0.041	0.031
Median	0.083	0.063
75th percentile	0.125	0.094
90th percentile	0.188	0.167

We assume that each hour of commute time has a utility cost estimated to be half of the hourly wage of workers (see Van Ommeren, Van den Berg, and Gorter (2000)). Hence, the total median cost for the median commuter should be 0.083/2 expressed as a fraction of the wage, or 0.083/2*(w/y) as a fraction of output (normalized to 1).

The total median cost is also calculated from the distribution of wage offers. Letting ρ^m be the median commute distance, $ln2/\alpha$, the total cost incurred for the median commuter is therefore given by

$$0.083/2 * (w/y) = \tau\rho^m = \ln 2/\alpha$$

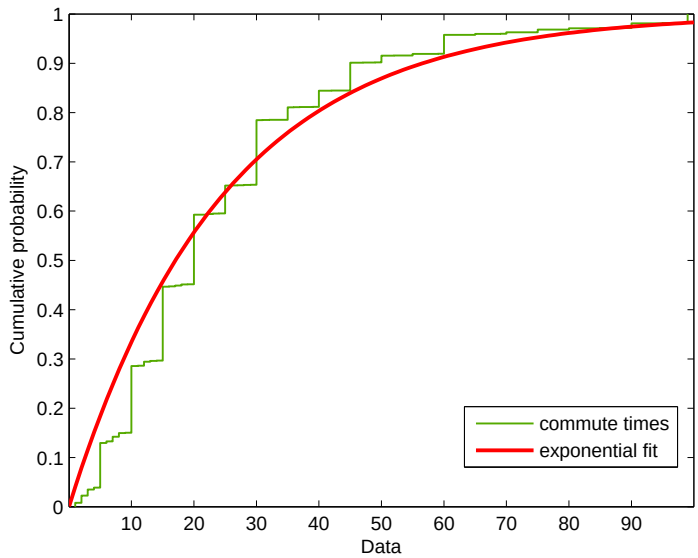
or

$$\tau\alpha = \frac{\ln 2}{0.083/2 * (w/y)}.$$

We then estimate α from the slope of the distribution F in the data. Inspection of Figure 5 shows that there is an optimal value of α that best approximates the c.d.f. We find empirically that it is equal to 9.77 after estimating ln(1-F($\alpha\rho$))=$\alpha\rho$ from the data. Hence, $\tau = 0.52$.

Unsurprisingly, given that commute costs per kilometer are higher in France and the benefits are higher, the mean commute time as well as the median are two percentage points lower in France: the unemployed are more choosy.

Figure 5: Distribution of Commute Times in the U.S.



The program finds the parameters of the model given a target unemployment rate of 4.2% in the U.S. (the average between March 1999 and March 2000), and a target job hiring rate of $p = 1/2.4$ monthly. The latter implies an average duration of unemployment of 2.4 months and therefore imposes a value for σ given that $u = \sigma/(\sigma + p)$ then $\sigma = p(u/(1-u)) = 0.0183$.

We match the mobility rate to a target value of 17.2% annually with (Eq. 15). The program finds the values of α and λ_H that are consistent with the target for mobility, given ρ^U, obtained from (Eq. 6).

We set $p(\theta) = A\theta^{0.5} * F(\rho^U)$. Setting $\theta = 1$ gives $A = 0.586$. Together with the free-entry condition, (Eq. 8), this fixes a value for recruiting costs c after normalizing $y = 1$.

To find the cost of commuting in Europe relative to the US, we use information from the tax authorities in the U.S. and France as well as gas prices given in Table 3. The Internal Revenue Service (IRS) in the U.S. and the tax authority BO Impôts in France provide standard mileage rates when using a car for business. For 2007, the allowance was \$0.485 per mile (0.20 euro per

mile) in the U.S. and 0.514 euro per kilometer for a 6CV car.[11] In addition, the price of gas in most European countries is at least double that in the U.S. However, since the typical US car consumes more gas per kilometer than in Europe, we assume that $\tau_{EU} = 1.5^* \tau_{US}$.

6.4 Findings

The findings for the benchmark economy are given in Table 5. The second column of the table increases benefits from 0.25 to 0.4. The third column then increases taxes to 0.4, keeping benefits at 0.4. The fourth column increases the commute cost by 1.5, holding fixed the taxes and benefits in the earlier columns. The fifth column divides the arrival rate of housing offers by 2.1 (to match the mobility rate in Europe), keeping fixed the parameters in the other columns. The fact that the arrival of housing offers needs to be divided by 2.1 suggests that the housing market in Europe is considerably more sclerotic than that of the U.S.

The combination of benefits and taxes more than doubles the unemployment rate, similar to that in Mortensen and Pissarides (1999). The inclusion of the housing market frictions and commute increases the unemployment rate by about 50%, from 9.6% to 13.3%.

It is also possible to calculate elasticities and slopes implied by the model. In particular, we can examine how changes in benefits, taxes, housing frictions and commuting costs affect unemployment and mobility. The elasticities and slopes are given in Table 6.

Table 5: U.S. Calibration

	Benchmark	Higher b	Higher b, tax	$\tau * 1.5$	$\lambda_h/2.1$
θ_h	1.000	0.685	0.268	0.194	0.186
ρ^U	0.055	0.051	0.043	0.036	0.036
Unemp	0.042	0.054	0.096	0.129	0.133
Un. Dur.	2.400	3.125	5.802	8.015	8.336
Reject	0.836	0.848	0.869	0.889	0.891
Mobility (x 100)	0.244	0.227	0.197	0.169	0.082

[11] The rate is progressive with power in France, ranging from 0.37 to 0.67 euro per kilometer.

Table 6: Elasticities and Slopes

	Unemployment		Mobility	
	Elasticity	Slope (%)	Elasticity	Slope (%)
Benefits	0.505	0.839	−0.066	−0.076
Taxes	0.270	0.511	−0.069	−0.090
Commuting cost (τ)	0.698	0.558	−0.330	−0.184
Housing (λ)	−0.093	−0.903	0.964	6.543

Our findings indicate that labor market institutions account for a large part of cross-country differences in unemployment but perform poorly in terms of explaining low mobility. Adding in housing frictions and commute costs delivers both low mobility and a quite sizeable increase in unemployment. Taxes and benefits alone generate a 4 percentage point increase in unemployment when European values are chosen instead of US values, a realistic increase in commute costs of 50% and a 30% increase in frictions in the housing markets raise unemployment by an additional 4 to 5 percentage points. Interestingly, housing frictions, per se, account for only a small portion of unemployment when commute costs are low. In other words, there is strong complementarity between the two parameters.

6.5 Robustness

Due to the fact that the wage is not endogenous we now provide several robustness exercises to show how the findings change with a change in the parameters. Table 7 shows how unemployment is affected by changes in β, ε and α. Changing β or α has only small effects on unemployment. However, changes in ε has large effects on unemploymeng. When $\varepsilon = .15$ unemployment rises to over 20%. Note that this value of ε means that the wage of the worker falls by 85%.

Table 7: Robustness: Effects on Unemployment

	Benchmark	Higher b	Higher b, tax	$\tau * 1.5$	$\lambda_h/2.1$
Changes in β					
$\beta = 0.5$	0.042	0.054	0.0956	0.1272	0.1322
$\beta = 0.4$	0.042	0.054	0.0911	0.1212	0.1263
$\beta = 0.3$	0.042	0.054	0.0877	0.1166	0.1219
$\beta = 0.6$	0.042	0.054	0.1013	0.1349	0.1399
$\beta = 0.7$	0.042	0.054	0.1088	0.1449	0.1499
Changes in ε					
$\varepsilon = 0.15$	0.042	0.0597	0.1475	0.197	0.2026
$\varepsilon = 0.25$	0.042	0.0564	0.1124	0.1498	0.1551
$\varepsilon = 0.35$	0.042	0.054	0.0956	0.1272	0.1322
$\varepsilon = 0.45$	0.042	0.0522	0.0861	0.1143	0.1193
$\varepsilon = 0.55$	0.042	0.051	0.0805	0.1068	0.1118
Changes in α					
$\alpha = 9.77$	0.042	0.054	0.0956	0.1272	0.1322
$\alpha = 11$	0.042	0.0539	0.0953	0.1267	0.1317
$\alpha = 13$	0.042	0.0538	0.095	0.1259	0.1309
$\alpha = 9$	0.042	0.054	0.0957	0.1275	0.1325
$\alpha = 7$	0.042	0.0541	0.0961	0.1282	0.1334

7 Concluding Comments

In this paper we have taken seriously the idea that labor market frictions, and in particular the reservation strategies of unemployed workers when they decide whether to accept a job offer, depend strongly on the functioning of the housing market. This interconnection between two frictional markets (housing and labor) can be used to understand differences in the functioning of labor markets. This paper has offered such a model, based on decisions to accept or reject a job offer, given the commuting distance to jobs. The model is relatively parsimonious, thanks to simplifying assumptions such as the isotropy of space, an unrealistic assumption but which conveniently provides closed form solutions and makes it possible to explain quit, job acceptance and geographic mobility decisions with a decision rule based on a single dimension.

We have attempted to explain differences in mobility rates between Europe and the US. A calibration exercise of "Europe" and the US is able to account for differences in unemployment and mobility thanks to a parameter which captures the speed of arrival of housing offers to households.

In our calibration, we find that labor market institutions account for a large part of cross-country differences in unemployment but perform poorly in terms of explaining low mobility. In contrast, our "spatial block", that is housing frictions combined to higher commute costs, explain well low mobility and a quite sizeable increase in unemployment : while taxes and benefits generate a 4 percentage point increase in unemployment when European values are chosen instead of US values, a realistic increase in commute costs of 50% and a 30% increase in frictions in the housing markets raise unemployment by an additional 4 to 5 percentage points.

Interestingly enough, housing frictions, per se, account for only a small portion of unemployment when commute costs are low: there is a strong complementarity between the two parameters: when commuting is costly and when it is difficult to relocate in the future, then job rejection is much more frequent.

Future work should attempt to enrich the model to introduce more specific urban features such as anisotropy of space and the existence of centers in cities and suburbs, as well as different groups of the labor force. Our work is a first step in integrating housing and labor markets in a coherent

macroeconomic model. In particular, since the model is simple, it can be extended to deal with new issues such as discrimination in the housing market, mobility allowances or "moving toward opportunity" schemes, spatial misatch issues and so on, as in the urban economics literature.

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Table 8: Mobility, Regulations and Unemployment

	Mobility rate outside the area within 3 yrs.	Housing market regulation index	Average unemployment rate 1995-2001
Denmark	0.054	3.6	5.3
Netherlands	0.029	3	4.2
Belgium	0.013	3.17	8.5
France	0.042	3.6	10.4
Ireland	0.010	3.2	7.9
Italy	0.011	4.24	10.7
Greece	0.011	4.31	10.5
Spain	0.009	4.81	14.5
Portugal	0.007	4.54	5.6
Austria	0.015	3.62	4.02
Finland	0.058	2.53	11.9
Germany	0.021	3.76	8.3
UK	0.072	2.22	6.5

Source: Mobility data from ECHP. Regulation indices from Djankov et al. (2002). Unempoyment rates from Eurostat.

8 Appendix

8.1 Evidence on Mobility, Regulations and Why People Move

This section provides evidence on the relationship between housing market regulations and mobility across the EU. Table 8 shows an index of housing market regulations derived by xxxx and unemployment rates for several countries.

Table 9 gives some additional information on reasons for moving.[12]

Finally, Table 10 shows that the reasons for work-related mobility are quite different within and across counties. As expected, intra-county work-related moves bring workers closer to their job, while 70% of inter-county work-related moves are related to a new job.

[12] Why People Move: Exploring the March 2000 Current Population Survey, Current Population Reports, P23-204, May, 2001, U.S. Department of the Census.

Table 9: Reasons for Moving, U.S.

Work		**House**	
new job or job transfer	60.5%	wanted own home, not rent	22.2%
look for work or lost job	9.7%	wanted new or better home/apartment	35.8%
closer to work / easier commute	19.6%	wanted better neighborhood/less crime	8.6%
retire	2.4%	wanted cheaper housing	10.8%
other job related reason	7.7%	other housing reason	22.6%
All work related reasons	100%	All house related reasons	100%
Family		**Other**	
change in marital status	23%	attend or leave college	38.2%
establish own household	27.2%	change of climate	11.0%
other family reason	49.8%	health reason	17.2%
All family related reasons	100%	other reasons	33.6%
		All other related reasons	100%

Table 10: Work-related Moves, U.S.

Work related	All	Intra	Inter
new job or job transfer	60.5%	24.5%	69.4%
look for work or lost job	9.7%	9.4%	7.7%
closer to work / easier commute	19.6%	53.5%	13.6%
retire	2.4%	1.4%	3.0%
other job related reason	7.7%	11.1%	6.3%
total	100%	100%	100%

8.2 Link between F, F_J and G_S

We start from the integrand $A = \int\int \max[U, E(\rho'), E(\rho^{''})] dF_J(\rho') dG_S(\rho'')$. We can rewrite A as

$$\begin{aligned} A &= \int\int \{I[\rho' > \rho_U] I[\rho'' > \rho_U] U \\ +I[\rho' &< \rho''] I[\rho' < \rho^U] E(\rho') \\ +I[\rho'' &< \rho'] I[\rho'' < \rho^U] E(\rho'')\} \\ & dF_J(\rho') dG_S(\rho''). \end{aligned}$$

This can be written as:

$$\begin{aligned} A &= U \int_{\rho^U} dF_J(\rho') \int_{\rho^U} dG_S(\rho'') \\ &+ \int_0^{\rho^U} E(\rho') \left(\int_{\rho'} dG_S(\rho'') \right) dF_J(\rho') \\ &+ \int_0^{\rho^U} E(\rho'') \left(\int_{\rho''} dF_J(\rho') \right) dG_S(\rho''), \end{aligned}$$

or

$$\begin{aligned} A &= U(1 - F_J(\rho^U))(1 - G_S(\rho^U)) \\ &+ \int_0^{\rho^U} E(\rho') \left(1 - G_S(\rho')\right) dF_J(\rho') \\ &+ \int_0^{\rho^U} E(\rho'') \left(1 - F_J(\rho'')\right) dG_S(\rho''). \end{aligned}$$

Letting $F = 1 - (1 - F_J(\rho))(1 - G_S(\rho))$, gives

$$dF = (1 - F_J) dG_S + (1 - G_S) dF_J,$$

and we can thus rewrite as

$$\begin{aligned} A &= U(1-F_J(\rho^U))(1-G_S(\rho^U)) \\ &\quad + \int_0^{\rho^U} E(\rho)dF(\rho) \\ &= \int \max(U, E(\rho))dF(\rho) \end{aligned}$$

The last equality comes from the observation that $1-F(\rho^U) = (1-F_J(\rho^U))(1-G_S(\rho^U))$.

8.3 Proofs

Proof the slope of E(ρ)

We need to rewrite Eq. 1 and Eq. 2 as:

$$\begin{aligned} (r+s)E(\rho) &= w - \tau\rho + sU + \lambda_H \int_0^{\rho^U} [E(\rho') - E(\rho)]dG_N(\rho') \qquad (18) \\ &\quad + \delta \int_0^{\rho^u} [E(\rho') - E(\rho)]dG_S(\rho'') + \delta \int_{\rho^u}^{+\infty} [U - E(\rho)]dG_S(\rho'') \\ (r+p)U &= b + p\int_0^{\rho^u} [E(\rho')]dF(\rho') + pU(1-F(\rho^u)). \qquad (19) \end{aligned}$$

Proof of the determination of ρ^U

Rewrite the Bellman equations as:

$$\begin{aligned} rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int_0^{\rho^E(\rho)} (E' - E)dG_N(\rho') \\ &\quad + \int_0^{\rho^U} [E''(\rho) - E(\rho)]dG_S(\rho) + \int_{\rho^U}^{+\infty} [U - E(\rho)]dG_S(\rho) \\ rU &= b + p\int_0^{\rho^U} (E' - U)dF(\rho') \end{aligned}$$

Here, since $\rho^E(\rho) = \rho$, we apply this formula in $\rho = \rho^U$ of E in ρ^U, we have:

$$\begin{aligned} rE(\rho^U) &= w - \tau\rho^U + s(U - E(\rho^U)) + \lambda_H \int_0^{\rho^U} (E' - E(\rho^U))dG_N(\rho') + \int_0^{\rho^U} [E''(\rho) - E(\rho)]dG_S(\rho) \\ &= rU = b + p\int_0^{\rho^U} (E' - U)dF(\rho') \end{aligned}$$

so

$$\rho^U = \frac{\lambda_H}{\tau}\int_0^{\rho^U}(E'-U)dG_N(\rho') + \frac{\delta}{\tau}\int_0^{\rho^U}[E''(\rho)-E(\rho)]dG_S(\rho) - \frac{p}{\tau}\int_0^{\rho^U}(E'-U)dF(\rho') + \frac{w-b}{\tau} \tag{20}$$

This equation simplifies a bit. Noting that

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r+s+\lambda_H G_N(\rho)+\delta}$$

integration by parts leads to

$$\int_0^{\rho^U}(E'-U)dH(\rho') = \int_0^{\rho^U}\frac{\tau H(\rho)}{r+s+\lambda_H G_N(\rho)}d\rho$$

for all H distributions such that $H(0)=0$, we can thus rewrite ρ^U as in the text (equation Eq. 6).

Proof of Proposition 1

Fully differentiating ρ^u gives

$$d\rho^U\left(\frac{r+s+pF(\rho)}{r+s+\lambda_H G_N(\rho)+\delta}\right) = \left(\int_0^{\rho^U}\frac{G_N(\rho)(r+s)+pF(\rho)G_N(\rho)}{[r+s+\lambda_H G_N(\rho)+\delta]^2}d\rho\right)d\lambda_H \tag{21}$$
$$-\left(\int_0^{\rho^U}\frac{F(\rho)d\rho}{r+s+\lambda_H G_N(\rho)+\delta}\right)dp + d\left(\frac{w-b}{\tau}.\right)$$

Equation (Eq. 21) indicates that ρ^U depends positively on λ_H, negatively on p and positively on $w-b$. An increase in λ_H (more housing offers) will shift the curve in Figure 2 upward, raising ρ^U and thus the acceptance rate of the unemployed. The intuition is simply that they take the job offer knowing that they will be able to find a better location in the near future because housing offers arrive very frequently.

Proof of Proposition 3

Differentiating u, and letting ω_θ represent the partial derivative of θ to λ_H. gives:

$$\begin{aligned}\frac{du}{d\lambda_H} &= \frac{-\delta g_S(\rho^U)\omega_\rho(\sigma+p(\theta)F(\rho^U)) - \sigma[-\delta g_S(\rho^U)\omega_\rho + p(\theta)f(\rho^U)\omega_\rho + p'(\theta)F(\rho^U)\omega_\theta]}{[\sigma+p(\theta)F(\rho^U)]^2} \\ &= \frac{-\delta g_S(\rho^U)p(\theta)F(\rho^U)\omega_\rho - \sigma\left[p(\theta)f(\rho^U)\omega_\rho + p'(\theta)F(\rho^U)\omega_\theta\right]}{[\sigma+p(\theta)F(\rho^U)]^2}.\end{aligned}$$

Next, note that

$$\frac{d\sigma}{d\lambda_H} = -\delta g_S(\rho^U)\frac{\partial \rho^U}{\partial \lambda_H} = -\delta g_S(\rho^U)\omega_\rho < 0,$$

where ω_ρ is simply a convenient notation for the partial derivative of ρ^u to λ_H and

$$\begin{aligned}\frac{d[p(\theta)F(\rho^U)]}{d\lambda_H} &= p'(\theta)F(\rho^U)\frac{\partial \theta}{\partial \lambda_h} + p(\theta)f(\rho^U)\frac{\partial \rho^U}{\partial \lambda_H} \\ &= p(\theta)f(\rho^U)\omega_\rho + p'(\theta)F(\rho^U)\omega_\theta > 0.\end{aligned}$$

8.4 Adding up moving costs

Let C be a relocation cost paid by workers. We ignore here G_S assumed to be degenerate and fix $\delta = 0$. We have thus:

$$\begin{aligned} rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int \max\left[0, (E' - E - C)\right] dG_N(\rho') \\ rU &= b + p\int (E' - U) dF(\rho') \end{aligned}$$

E is downward sloping in ρ with slope

$$\frac{dE}{d\rho} = \frac{-\tau}{r + s + \lambda_H P_W}$$

where $P_W \geq 0$ is the probability to move conditional on receving a housing offer, with $0 < P_W < 1$ possibly depends on ρ. The function $E(\rho)$ is thus monotonic and there is thus a well-defined reservation strategy, with a reservation distance denoted by ρ^E for the employed above which a **housing offer** is rejected. Note in addition that there is NOW state-dependence in the reservation strategy of the employed: we have that $\rho^E(\rho)$ with presumably $d\rho^E/d\rho > 0$: the further away the tenants live from her job, the less likely they will reject an offer. (NB: to be shown in the general case). We can rewrite

$$P_W = G_N(\rho^E(\rho))$$

and obtain the Bellman equations as:

$$\begin{aligned} rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int_0^{\rho^E(\rho)} (E' - E - C) dG_N(\rho') \\ rU &= b + p \int_0^{\rho^U} (E' - U) dF(\rho') \end{aligned}$$

8.5 Housing Market Regulations and Housing Offers

We now provide a simple explanation for the link between housing market regulations mentioned in the Introduction and the parameter λ_H, the arrival of housing offers, that is, we describe an extension that endogenizes λ_H.

Landlords post vacancies and screen applicants. They offer a lease to the "best applicants", in a sense defined below. In case of a default on the rent by the tenant, however, landlords incur a loss due to the length of litigation and eviction procedures. The asset value of owning a dwelling with a tenant defaulting on the rent is denoted by Λ. Therefore, Λ will decline with more regulations in the rental housing market, due to the length of the procedures to recover the unpaid rent and the dwelling.

Potential tenants (applicants) are all ex-ante homogenous but ex-post may represent a default risk for the landlord. More precisely, at the time of the contact between the applicant, the landlord gets a random signal on the tenant and postulates that the particular applicant has a specific default rate δ_i (a Poisson rate in continuous time). It follows that for such a tenant, the value of a filled vacancy for the landlord is:

$$rH_F = R + \delta_i(\Lambda - H_F),$$

where R is the rent and H_F is the value of a filled vacancy.[13] Therefore,

$$H_F = \frac{R + \delta_i \Lambda}{r + \delta_i}. \tag{22}$$

[13] Recall that we assumed in the model that the flow of services of the dwelling to the tenant was exactly compensating the rent so we did not need to include the rent in the Bellman equations of workers. We also assume that workers do not benefit from defaulting on the rent: in case of default, they may have a disutility exerted by landlords or their lawyers so that, after default, the value of being in a dwelling is not higher than it was when paying. Therefore, the expression for the Bellman equation of tenants derived previously is unchanged.

The derivative of H_F with respect to δ_i is given as $\Lambda r - R$, which is negative since the landlord's value of a default, Λ, cannot be higher than the capitalized value of the rent R/δ_i. Therefore, H_F is decreasing in δ_i, from R/r to Λ. This implies that there will be a well defined reservation strategy by landlords given the signal they receive.

To derive this reservation strategy, we denote by H_V the value of a vacant housing unit prior to screening. This value is exogenous and is given, in the long-run, by the cost of construction of new housing units. Therefore, it is independent of rental regulations and, in particular, of Λ. At the time of contact with a tenant, landlords decide to offer a lease if the perceived value of default δ_i is below a cut-off value $\overline{\delta}$ with

$$\begin{aligned} \frac{R+\overline{\delta}\Lambda}{r+\overline{\delta}} &= H_V \\ \text{or } \overline{\delta} &= \frac{R-r\Lambda}{H_V-\Lambda} \end{aligned}$$

Note that $\overline{\delta}$ is increasing in Λ: the higher is Λ, the easier it is to accept a tenant since the risk of default is lower.

Hence, the screening rate, α, of tenants is prob $(\delta_i > \overline{\delta})$. Denoting by $L(\delta_i)$ the c.d.f. of default rates, we have

$$\alpha = 1 - L\left(\frac{R-r\Lambda}{H_V-\Lambda}\right)$$

The screening rate is therefore increasing when Λ is lower, that is, when rental housing market regulations are higher.

Finally, denote by ϕ the Poisson rate at which tenants receive dwelling offers. We assume that this contact rate is exogenous. It then follows that $\lambda_H = \phi\alpha$. Therefore,

Proposition 4: *λ_H is decreasing in the amount of housing market regulations.*

8.6 Endogenous Wage: A Wage Posting Model

The main equations of the model are unchanged, that is, Eq. 1 and Eq. 2 lead to the reservation rule similar to Eq. 6, except that it now depends on the wage,

$$\rho^U(w) = \frac{w(1-t)-b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) - pF(\rho)}{r+s+\lambda_H G_N(\rho)+\delta} d\rho. \tag{23}$$

The firm maximizes the ex-ante value of a vacancy,

$$w = ArgMax\, V(w) = \frac{-\gamma + q(\theta)F(\rho^U(w))J(w)}{r}.$$

The first order condition is given as

$$q(\theta)f(\rho^U)\frac{d\rho^U}{dw}[y-w] = 1. \tag{24}$$

Eq. 24 states that the marginal gain of raising the wage is equal to the marginal increase of accepted job offers, $f(\rho^U)\frac{d\rho^U}{dw}$, times the probablility per unit of time to meet a worker, $q(\theta)$, times the value of a job, $J(w)$, has to equal to the loss of profits of higher wages, $-1/(r+\sigma)$.

Differentiating ρ^U from Eq. 23, we obtain

$$\frac{d\rho^U(w)}{dw} = (1-t)A(p), \tag{25}$$

with

$$A(p) = \frac{1}{\tau}\frac{1}{1 - \frac{\lambda_H G_N(\rho^U)+\delta G_S(\rho^U)-pF(\rho^U)}{r+s+\lambda_H G_N(\rho^U)+\delta} - \int_0^{\rho^U} \frac{[\lambda_H G_N(\rho)+\delta G_S(\rho)-pF(\rho)]\delta g_S(\rho^U)}{[r+s+\lambda_H G_N(\rho)+\delta]^2} d\rho},$$

which is downward sloping. Substituting Eq. 25 into Eq. 24, we obtain

$$q(\theta)f(\rho^U)(1-t)A[y-w] = 1,$$

and therefore an expression for the optimal wage; in particular, the net wage and the gross wage:

$$\text{Gross wage} \quad : \quad w = y - \frac{1}{q(\theta)f(\rho^U)A(1-t)}$$
$$\text{Net wage} \quad : \quad w(1-t) = y(1-t) - \frac{1}{q(\theta)f(\rho^U)A}.$$

The inclusion of taxes raises the gross wage and reduces the net wage. Finally, replacing the endogenous value of the wage into Eq. 23, we obtain a value for ρ^U which is downward sloping in p and therefore θ.

$$\rho^U = \frac{\left[y(1-t) - \frac{1}{q(\theta)f(\rho^U)A}\right] - b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) - pF(\rho)}{r+s+\lambda_H G_N(\rho) + \delta} d\rho. \tag{26}$$

The free-entry of firms given their optimal wage is

$$V(w^*) = 0$$

leading to

$$\frac{y-w}{r+\sigma} = \frac{c}{q(\theta)F(\rho^U)},$$

Now, replacing for the wage, we obtain

$$\frac{1}{r+\sigma}\frac{1}{A(p)(1-t)} = c\frac{f(\rho^U)}{F(\rho^U)} \tag{27}$$

We assume here that the distribution f has the declining likelihood ratio property, that is $f(x)/F(x)$ is decreasing in x. It follows that the right-hand side is decreasing in ρ^U and $A(p)$ being decreasing in p, the right-hand side is increasing in p and θ.

Overall, equation Eq. 27 defines a new relation between ρ^U and θ, which is negatively sloped.