How Amenities Affect Job and Wage Choices over the Life Cycle*

Ed Nosal
Federal Reserve Bank of Cleveland
and
Peter Rupert
Federal Reserve Bank of Cleveland

June 2006

Abstract

Job amenities are explicitly included in a model of job choice over the life cycle. The amenities are characterized by an indivisibility–a worker must be present at a job to enjoy its amenities. This characterization has implications on initial job choice, a worker's wage profile and whether they move to a higher or lower paying job.

Keywords: job changes, amenities, lifetime wage profile

*We thank Rob Shimer, Richard Rogerson, Audra Bowlus, Chris Robinson, Todd Stinebrickner, Randy Wright, participants at the SED meetings 2002, NBER Summer Institute, the University of Western Ontario and the Editor, Associate Editor and anonymous referees. Contact: Rupert: prupert@clev.frb.org. The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or of the Board of Governors of the Federal Reserve System.

1 Introduction

It is well known that a job consists of many characteristics valued by the worker, only one of which is the wage. In equilibrium, the pricing of these characteristics leads to observed differences in wages—a compensating differential—such that the value of jobs is equalized. This topic has generated a considerable amount of research focusing on the existence, or extent, of such differentials.

In this paper, job amenities are explicitly included in a model of job choice over the life cycle. The amenities are characterized by an indivisibility–a worker must be present at a job to enjoy its amenity. This characterization has implications on initial job choice, a worker's wage profile and whether they move to a higher or lower paying job.

A life cycle model is constructed that allows workers to choose their career path over various jobs, where a job is defined by a wage and amenity bundle. We find that when amenities and wages are not perfect substitutes, a worker will always want to change jobs at some point in life. The basic intuition underlying this result is that workers can smooth their amenity consumption over their lifetimes only by changing jobs, this arises from the assumption of the indivisibility of amentities. When amenities and wages are perfect substitutes, workers will change jobs only if a smoother *total* consumption path—where total consumption is the sum of the amenity and a market good—is obtained by a job change.

Here, when a worker changes jobs, the wage will necessarily change. Our model makes rather stark predictions concerning wages over the life cycle. In particular, some workers initially choose high paying jobs and then migrate to lower paying ones; while other workers will follow precisely the opposite strategy. What determines the sequencing of jobs over time, in terms of migrating to higher or lower paying jobs, is the worker's rate of time preference in relation to the market interest rate. Workers who discount the future at a higher rate than the market interest rate will move to higher paying jobs over their lifetimes, while those who discount at a lower rate than the market interest rate will move from higher to lower paying jobs.

The voluntary movement of workers from high to low paying jobs is consistent with the data. Using the Panel Study of Income Dynamics it is possible to identify workers who have changed employers *voluntarily*.[1] While the majority of the voluntary leavers move to new jobs that pay more than their previous job, a surprisingly substantial proportion (approximately 42%) move to new jobs where the wages are actually *lower*.[2] This paper provides a model that can account for workers moving from either high to low or low to high paying jobs where the direction of the movement depends upon the worker's (relative) rate of time preference.

The closest paper to ours is Rosen (1972). In his paper, jobs offer different amounts of general training, or learning opportunities. Jobs that provide more (on-the-job) training pay lower wages; higher paying jobs require higher investments in general training. Over the life cycle, workers change jobs, moving from lower paying jobs to higher paying ones. A key prediction that is absent from Rosen (1972) model, but is in the data, is that workers move from high to lower paying jobs.[3]

Finally, Hwang, Mortensen, and Reed (1998) extend the Mortensen (1990) basic job search model to include a non-wage component. As in the Rosen (1972) model, there is a distribution across firms of the cost of providing the amenity. Hwang *et al.* (1998) show that adding search into a model with a non-wage component can give rise to a bias in empirical models that try to uncover the marginal willingness to pay for amenities. In other words, although workers are willing to trade off wages for the amenity, the search model can generate a positive relation between the two.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes the case where wages and amenities are not perfectly substitutable within a job, while section 4 analyzes the case where they are perfect substitutes. Section 5 presents some stylized facts from the PSID associated with employer changes by workers. These data help to both motivate and bring perspective to the model. Section 6 concludes.

[1] Although in principle it is difficult to know whether a separation is voluntary or involuntary, the question in the PSID asks workers to choose from several reasons as to why they left their last job, one of which being that they chose to leave.

[2] The model makes no distinction between employer changes or job changes, though in the PSID the question concerns employer changes. However, in this paper, job and employer changes are used interchangeably.

[3] Rosen (1972) has workers changing jobs, but in effect has linear preferences as workers only maximize the present discounted value of lifetime income.

2 The Model

Workers are born at date 0 and live for one period of continuous time. At each instant in time workers inelastically supply one unit of labor to a job. For simplicity, it is assumed that there are two types of firms or *jobs*, job 1 and job 2. We show below that this can be easily generalized to n jobs. Each job $i \in 1,2$ is characterized by a wage, w_i, and a fixed level of the amenity, A_i. There is no uncertainty over the wage/amenity package at each job. There is free entry and exit of firms for each type. The production function for each firm type uses labor as its only input and is characterized by the marginal product of labor being equal to the average product of labor. Competition and free entry of each firm type implies that the wage at each job, w_i, is equal to the marginal product. At each time t the worker decides where to work. The indicator function $\alpha(t)$ describes the worker's job choice at date t. In particular, if $\alpha(t) = 1$, then the worker chooses job 1 at date t; if $\alpha(t) = 0$, then the worker chooses job 2 at date t.

Agent's preferences are represented by the momentary utility function $u(c(t), A_i)$, where $c(t)$ represents consumption of a private good at time t and A_i is the job amenity that the worker consumes at date t. Agents discount the future at rate δ and can borrow and lend at interest rate r. The worker's total savings (or stock of wealth) at date t is denoted by $a(t)$. The instantaneous change in the worker's wealth at date t is given by the sum of (i) interest income on existing wealth, $ra(t)$, and (ii) the difference between the date t wage, w_i, $i \in \{1,2\}$, and consumption at date t, $c(t)$. Below, an explicit structure on the momentary utility function will be imposed that will reflect the assumed substitutability between wages and amenities.

In order to motivate the incentive to change jobs and to make the worker's decision problem interesting, we assume that $w_1 > w_2$ and $A_1 < A_2$.

3 Wages and Amenities are not Perfectly Substitutable

This section specifies a particular functional form for preferences, where the wage alone does not embody all of the relevant aspects of the job. The specification assumes that within-job substitutability between the private good, c, and the amenity, A_i, is not perfect. A momentary utility function for a worker choosing job i that embodies this notion is given by:

$$u(c,A_i) = \ln(c) + A_i, \quad i = 1,2. \tag{1}$$

As will be seen below, with this specification of preferences, private consumption is independent of the level of amenity consumption in the sense that, for a given level of lifetime income, the optimal path of private consumption will not depend on the path of amenity consumption.

The worker will choose a private consumption stream, $\{c(t)\}$, and where to work, $\{\alpha(t)\}$, in a manner that solves:

$$\max_{\{c(t),\alpha(t)\}} \int_0^1 e^{-\delta t}[\ln(c(t)) + \alpha(t)A_1 + (1-\alpha(t))A_2]dt, \tag{2}$$

subject to

$$\dot{a}(t) = a(t)r + \alpha(t)w_1 + (1-\alpha(t))w_2 - c(t). \tag{3}$$

and

$$a(0) = a(1) = 0 \tag{4}$$

The objective function, (2), is the worker's lifetime utility. Equation (3) describes how the worker's wealth evolves over time. The equations contained in (4) simply says that the worker begins life with no wealth and (optimally) ends life with no wealth.

The current value Hamiltonian, $\mathscr{H}$, associated with the maximization problem (2)-(4) is,

$$\begin{aligned} \mathscr{H} &= \ln(c(t)) + \alpha(t)A_1 + (1-\alpha(t))A_2 + \\ &\quad \lambda(t)(a(t)r + \alpha(t)w_1 + (1-\alpha(t))w_2 - c(t)). \end{aligned} \tag{5}$$

The solution to the worker's problem is given by,

$$\mathscr{H}_{c(t)} = \frac{1}{c(t)} - \lambda(t) = 0, \tag{6}$$

$$\mathscr{H}_{\alpha(t)} = \begin{cases} A_1 - A_2 + \lambda(t)(w_1 - w_2) > 0 & \text{if} \quad \alpha(t) = 1 \\ A_1 - A_2 + \lambda(t)(w_1 - w_2) < 0 & \text{if} \quad \alpha(t) = 0 \\ A_1 - A_2 + \lambda(t)(w_1 - w_2) = 0 & \text{if} \quad \text{indifferent} \end{cases} \tag{7}$$

and

$$\dot{\lambda}(t) = -\mathscr{H}_a + \lambda(t)\delta \quad \text{or} \quad \frac{\dot{\lambda}(t)}{\lambda(t)} = -(r-\delta). \tag{8}$$

Equations (6) and (8) imply that private consumption grows at the rate $r-\delta$, i.e.,

$$\frac{\dot{c}(t)}{c(t)} = r - \delta. \tag{9}$$

The shape of the private consumption profile is given by the sign of $r-\delta$: If $r-\delta>0$, then private consumption is strictly increasing over the worker's life; if $r-\delta<0$, then private consumption is strictly decreasing over the worker's life; if $r-\delta=0$, then private consumption is constant.

The worker's job choice, $\alpha(t)$, is determined by (7). Since $\lambda(t)=1/c(t)$, the worker's job choice can be simplified to

$$\alpha(t) = 1 \text{ if } c(t) < \frac{w_1-w_2}{A_2-A_1} \tag{10}$$

$$\alpha(t) = 0 \text{ if } c(t) > \frac{w_1-w_2}{A_2-A_1} \tag{11}$$

and

$$\alpha(t) = 0 \text{ or } \alpha(t) = 1 \text{ if } c(t) = \frac{w_1-w_2}{A_2-A_1} \tag{12}$$

Hence, if at date t the worker's level of private consumption is less than $\frac{w_1-w_2}{A_2-A_1}$, then at date t it is optimal for the worker to be at job 1, (inequality (10)); if at date t the worker's level of private consumption is greater than $\frac{w_1-w_2}{A_2-A_1}$, then at date t it is optimal for the worker to be at job 2, (inequality (11)).

A worker will stay at job 1 his entire life if condition (10) holds for all $t\in[0,1]$. In this situation the worker's lifetime budget constraint is given by

$$c(0)\int_0^1 e^{-\delta t}dt = w_1\int_0^1 e^{-rt}dt,$$

which implies that

$$c(0) = \begin{cases} w_1\left(\frac{e^{-r}-1}{e^{-\delta}-1}\right)\frac{\delta}{r} & \text{if } r\neq\delta \\ w_1 & \text{if } r=\delta. \end{cases}$$

Hence, the worker will stay at job 1 his entire life if

$$w_1 < \frac{w_1-w_2}{A_2-A_1} \text{ when } r=\delta, \tag{13}$$

$$w_1\left(\frac{e^{-r}-1}{e^{-\delta}-1}\right)\frac{\delta}{r} < \frac{w_1-w_2}{A_2-A_1} \text{ when } r<\delta, \tag{14}$$

$$w_1\left(\frac{e^{-r}-1}{e^{-\delta}-1}\right)\frac{\delta}{r}e^{r-\delta} < \frac{w_1-w_2}{A_2-A_1} \text{ when } r>\delta. \tag{15}$$

(The left-hand-side of each of the inequalities represents the maximum amount of private consumption that the worker would ever consume in any period t. Note that $\left(\frac{e^{-r}-1}{e^{-\delta}-1}\right)\frac{\delta}{r} > 1$ when $\delta > r$ and $\left(\frac{e^{-r}-1}{e^{-\delta}-1}\right)\frac{\delta}{r}e^{r-\delta} > 1$ when $r > \delta$.) For any of (13)-(15) to hold, it must be the case that the instantaneous value associated with job 1, $u(w_1) + A_1$, must be "sufficiently greater" than the instantaneous value associated with job 2, $u(w_2) + A_2$. To see this, note a linear approximation of $\ln(w_2)$ taken at w_1 can be rearranged to read

$$w_1 > \frac{w_1 - w_2}{\ln(w_1) - \ln(w_2)}, \tag{16}$$

and define $\varepsilon_1 > 0$ as

$$\ln(w_1) + A_1 \equiv \ln(w_2) + A_2 + \varepsilon_1. \tag{17}$$

(Definition (17) implies that the instantaneous value of job 1 is greater than that of job 2 since $\varepsilon_1 > 0$.) The right-hand-side of (16) can be rewritten in terms of amenities, i.e.,

$$\frac{w_1 - w_2}{\ln(w_1) - \ln(w_2)} = \frac{w_1 - w_2}{A_2 - A_1 + \varepsilon_1} < \frac{w_1 - w_2}{A_2 - A_1}.$$

From this we see that the greater is ε_1, the greater will $\frac{w_1-w_2}{A_2-A_1}$ be relative to $\frac{w_1-w_2}{\ln(w_1)-\ln(w_2)}$. For a large enough value of ε_1, $\frac{w_1-w_2}{A_2-A_1}$ can be made to be greater than w_1. Hence, if ε_1 is sufficiently large, i.e., the instantaneous value of job 1 is sufficiently larger than job 2, then inequalities (13)-(15) can hold, which means that a worker will spend his entire life at job 1.

Using the same kind of logic, a worker will spend his entire life at job 2 if the instantaneous value of job 2 is "sufficiently greater" than the instantaneous value of job 1. A worker will spend his entire life at job 2 if

$$w_2 > \frac{w_1 - w_2}{A_2 - A_1} \text{ when } r = \delta, \tag{18}$$

$$w_2\left(\frac{e^{-r}-1}{e^{-\delta}-1}\right)\frac{\delta}{r} > \frac{w_1 - w_2}{A_2 - A_1} \text{ when } r < \delta, \tag{19}$$

$$w_2\left(\frac{e^{-r}-1}{e^{-\delta}-1}\right)\frac{\delta}{r}e^{r-\delta} > \frac{w_1 - w_2}{A_2 - A_1} \text{ when } r > \delta. \tag{20}$$

If $\varepsilon_2 > 0$ is defined by $u(w_2) + A_2 \equiv u(w_1) + A_1 + \varepsilon_2$, and, since

$$w_2 < \frac{w_1 - w_2}{\ln(w_1) - \ln(w_2)}, \tag{21}$$

we have

$$\frac{w_1 - w_2}{\ln(w_1) - \ln(w_2)} = \frac{w_1 - w_2}{A_2 - A_1 - \varepsilon_2} > \frac{w_1 - w_2}{A_2 - A_1}.$$

Therefore, inequalities (18)-(20) will hold if $\varepsilon_2 > 0$ is sufficiently large, which means that a worker will spend his entire life at job 2.

A necessary, but not sufficient, condition for the worker to spend his entire life at job 1 is that

$$u(w_1) + A_1 > u(w_2) + A_2. \tag{22}$$

This condition is not sufficient because, as we will now demonstrate, a worker may optimally spend part of his life at both jobs 1 and 2 even when condition (22) holds. Let us now suppose that $\varepsilon_1 > 0$, defined in (17), is now "not too large" in the sense that

$$w_1 > \frac{w_1 - w_2}{A_2 - A_1}; \tag{23}$$

then the worker will *always* change jobs at least once. To see this suppose that the worker chooses job 1 and remains there for his entire life. From equation (9) there exists some t, say $\hat{t}$, where $c(\hat{t}) = w_1$; in addition (i) when $r = \delta$, $c(t) = w_1$ for all $t \in [0,1]$, (ii) when $r > \delta$, $c(t) > w_1$ for all $t \in [\hat{t}, 1]$, and (iii) when $r < \delta$, $c(t) > w_1$ for all $t \in [0, \hat{t}]$. Hence, inequality (23) in conjunction with (11), implies that the worker will *not* remain at job 1 for his entire life. By similar reasoning, if the instantaneous value of job 2 is greater than that of job 1, then if the difference between the values of these jobs is "not too large" the worker will not spend his entire life at job 2. In particular, we will say that $\varepsilon_2 > 0$ is sufficiently small if

$$w_2 < \frac{w_1 - w_2}{A_2 - A_1}. \tag{24}$$

Given that the worker will always change jobs when the difference between the values of jobs 1 and 2 is "not too large," the sequence of job choices will depend on the sign of $r - \delta$, as the following proposition describes.

Proposition 1 *If either $\varepsilon_1 > 0$ or $\varepsilon_2 > 0$ is not too large, i.e., either inequality (23) or (24) holds, respectively, then (i) if $r < \delta$, the worker changes jobs only once, moving from job 2 to job 1; (ii) if $r > \delta$, the worker changes jobs once, moving from job 2 to job 1; and (iii) if $r = \delta$ the worker can change jobs an arbitrary number of times and is indifferent as to which job to take at date $t = 0$.*

Proof. (i) When $r > \delta$, it is never optimal for the worker to move *from* job 2 *to* job 1. If the worker did follow this job sequence, then, by (10), it must be the case that private consumption falls after the job change. However, when $r > \delta$ the worker's optimal private consumption stream, implicitly given by (9), is always *strictly increasing* over time. This implies that the worker will move only from job 1 to job 2 and, therefore, can only change jobs at most once. The only possible equilibrium job choice strategy for the worker is to spend the first part of life at job 1 and the second part in job 2. This sequence of job choices is consistent with a strictly increasing lifetime private consumption profile, i.e., it is consistent with (10), (11) and (12).

(ii) When $r < \delta$, equation (9) implies that the worker's lifetime private consumption profile will be strictly decreasing over time. Hence, it is never optimal for the worker to change from job 1 to job 2, as this sequencing of job choices would not be consistent with a strictly decreasing private consumption profile, i.e., see (10) and (11). The equilibrium job choice strategy for the worker will be to spend the first part of life at job 2 and the second part at job 1; this sequence of job choice *is* consistent with a strictly decreasing lifetime private consumption profile and the worker will change jobs only once.

(iii) When the discount rate equals the interest rate, equation (9) implies that the worker's lifetime private consumption stream will be constant.[4] Since the worker changes jobs at least once and lifetime private consumption is constant, it must be the case that $c(t) = \frac{w_1 - w_2}{A_2 - A_1}$ for all $t \in [0,1]$. The worker's initial job choice and the number of job changes will now be characterized. The following notation turns out to be helpful. Define $D \equiv \int_0^1 e^{-rt} dt$ and $d_{t_i}^{t_j} \equiv \int_{t_i}^{t_j} e^{-rt} dt$, where $t_j > t_i$. One can interpret both D and $d_{t_i}^{t_j}$ in terms of "discounted time." That is, D represents the discounted value of one unit of time starting at $t = 0$; $d_{t_i}^{t_j}$ represents the discounted value of $t_j - t_i$ units of time t_i units of time from now. The present value of lifetime private consumption when $c(t) = \frac{w_1 - w_2}{A_2 - A_1}$ for all $t \in [0,1]$ is then simply $\frac{w_1 - w_2}{A_2 - A_1} D$. If the worker's initial job choice is, say, job 1, and changes

[4]The worker must change jobs at least once when $r = \delta$ and the difference between instantaneous utilities of the jobs is "sufficiently small." To see this suppose that the worker stays at job 1 for his entire life; then $c(t) = w_1$ for all t. Since the difference between instantaneous utilities is sufficiently small (and the worker chooses job 1 or job 2), inequality (23) must hold. But this contradicts inequality (10), which is the inequality that says the worker will stay his entire life at job 1.

jobs n times, where the last job is, say, job 2, then lifetime income is

$$w_1 d_0^{t_1} + w_2 d_{t_1}^{t_2} + w_1 d_{t_2}^{t_3} + \cdots + w_2 d_{t_n}^{1}, \tag{25}$$

Note that $\sum_{i=0}^{n} d_{t_i}^{t_{i+1}} = D$, where $t_0 \equiv 0$ and $t_{n+1} \equiv 1$, and that for a given r, D is just a number. The present value of lifetime income must equal the present value of lifetime private consumption, i.e.,

$$w_1(d_0^{t_1} + \cdots + d_{t_{n-1}}^{t_n}) + w_2(d_{t_1}^{t_2} + \cdots + d_{t_n}^{1}) = \frac{w_1 - w_2}{A_2 - A_1} D. \tag{26}$$

Hence, the worker spends $D_1 = d_0^{t_1} + \cdots + d_{t_{n-1}}^{t_n}$ units of discounted time at job 1 and $D_2 = d_{t_1}^{t_2} + \cdots + d_{t_n}^{1}$ units of discounted time at job 2. But, since $D_1 = D - D_2$, equation (26) is simply an equation in one unknown, D_1. Call the solution D_1^*. That is, D_1^* solves

$$w_1 D_1^* + w_2(1 - D_1^*) = \frac{w_1 - w_2}{A_2 - A_1} D \tag{27}$$

or

$$D_1^* = \frac{D}{A_2 - A_1} + \frac{w_2}{w_1 - w_2}. \tag{28}$$

Above, it was assumed that the worker's initial job choice was job 1, changed jobs n times and the last job was job 2. It turns out that it does not matter where the worker's initial job is, how many times he changes jobs or what his last job is; all that is required is that he spend the fraction D_1^*/D of discounted time in job 1 and the remainder in job 2. ■

To sum up, when the difference between instantaneous utilities of the two jobs is not too large, the sign of $r - \delta$ determines whether the worker moves from a high paying job to a low paying one or from the low paying job to a higher paying one. When $r > \delta$, the worker changes jobs once and moves from the high to low paying job. When $r < \delta$, the worker also changes jobs once but moves from the low to high paying job. When $r = \delta$ the worker will change jobs at least once and is indifferent between job 1 and job 2 as an initial job.

3.1 Discussion

3.1.1 Initial Job Choice

The intuition behind the choice of an initial job is easiest to see by fixing the amount of time spent in each job (assuming that individuals will work at both jobs). By choosing job 1, the higher wage job, first, lifetime income will be higher than if job 2 is chosen first; and, the higher the interest rate, r, the greater will be the difference in lifetime incomes. So, as the interest rate increases job 1 looks more and more attractive as a starting job. Conversely, job 2, as an initial job choice, will provide a higher lifetime value of amenities as will job 1 as an initial job choice; as the discount rate, δ, rises, the greater will be the difference in lifetime value of amenities. As a result, the higher the discount rate for a worker, the more attractive job 2 looks as a starting job.

When $r > \delta$ the "interest rate effect" associated with taking job 1 first dominates the "discount rate effect" of taking job 2 first. So, lifetime utility is higher when job 1 is chosen first. When $r < \delta$ the discount rate effect dominates the interest rate effect, leading to higher lifetime utility by choosing job 2 first. When $r = \delta$, the interest rate effect associated with taking job 1 first exactly offsets the discount rate effect associated with taking job 2 first, implying that the worker is indifferent between choosing job 1 and job 2 at date $t = 0$.

3.1.2 Why Do Workers Change Jobs?

In order to gain some intuition as to *why* individuals change jobs, assume that a worker lives for only an instant of time. As a first approximation, this allows us to ignore discounting.[5] Imagine that in this instant unit of time the worker spends a fraction q in job 1 and $(1-q)$ in job 2. Over this instant of time financial markets permit the worker to smooth consumption of the private consumption good, c, i.e., the worker can consume approximately $\bar{w} = qw_1 + (1-q)w_2$. But, of course, the worker is unable to smooth the consumption of the amenity since the amenity is job specific. Hence, if the worker smooths consumption of the private good, utility over the instant of time is (approximately) equal to $qu(\bar{w},A_1) + (1-q)u(\bar{w},A_2)$. If the worker spends the entire instant of time in either job 1 or job 2, i.e., the worker does not change jobs, then utility is equal to $\max\{u(w_1,A_1),u(w_2,A_2)\}$. For convenience suppose that $u(w_1,A_1) = u(w_2,A_2)$. The worker will

[5]Discounting is important in terms of explaining *which* job the worker will initially take but is not that important in terms of explaining *why* workers change jobs. For example, when $r = \delta = 0$, although the worker is indifferent between which job to take at date $t = 0$, he is not indifferent between changing and not changing jobs; he strictly prefers to change jobs.

prefer changing jobs, compared to staying in the same job, if

$$\begin{aligned} \ln(\bar{w}) + \bar{A} &> \ln(w_1) + A_1 = \ln(w_2) + A_2 \\ &= q(\ln(w_1) + A_1) + (1-q)(\ln(w_2) + A_2), \end{aligned} \tag{29}$$

where $\bar{A} = qA_1 + (1-q)A_2$. Since $\ln(\bar{w}) > q\ln(w_1) + (1-q)\ln(w_2)$, inequality (29) holds. Hence, workers want to change jobs because they effectively get to consume "the average" of bundles (w_1, A_1) and (w_2, A_2) and the only way to consume an average of the bundles is by changing jobs.[6]

3.1.3 Many Jobs

Except for the knife-edge case where $r = \delta$, workers will change jobs exactly once in their lifetimes. In reality, however, some workers may never change jobs or other workers may change jobs more than once over their lifetimes.

Suppose that instead of facing two possible job choices each worker is randomly given $n > 2$ jobs to choose from. Without loss of generality, let job 1 be the "best" job and job n be the "worst" job in the following sense,

$$u(w_1, A_1) \geq u(w_2, A_2) \geq \cdots \geq u(w_n, A_n). \tag{30}$$

If it turns out that the instantaneous utility of job 1 is substantially higher than job 2, then the worker chooses job 1 at date $t = 0$ and will never change jobs. The case considered in the body of the paper can be interpreted by having the instantaneous utilities of job 1 and job 2 not being significantly different from one another, but the instantaneous utility of job 2 substantially larger than job 3. In this situation, the worker will change jobs once: which job the worker chooses first will depend upon the sign of $r - \delta$. In general, if the instantaneous utilities associated with the first k jobs are not significantly different from one another, but there is a significant difference between the k^{th} and $k+1^{st}$ job, then the worker will change jobs k times. So by increasing the number of jobs available to workers and by relaxing the assumption that instantaneous utility of all jobs is equal, it is possible for the model to be consistent with observed outcomes in the data.

[6] Note that the inequality (22) is actually a statement about quasi-concavity. Recall that the notion of quasi-concavity is that a consumer can be made better off by consuming the average of two bundles that provide the same level of utility compared to consuming either one of the bundles. Even if $u(w_1, A_1) \neq u(w_2, A_2)$, but the difference between instanteous utilities is not too large, it will be the case that $\ln(\bar{w}) + \bar{A} > \{u(w_1, A_1), u(w_2, A_2)\}$.

3.1.4 Worker Heterogeneity

In the data, some individuals move from lower to higher paying jobs, while other individuals move from higher to lower paying jobs. The model can be made consistent with these observed facts if workers are heterogeneous. For example, one simple form of heterogeneity is that different workers have different discount rates. Let δ_i be the discount rate for worker i. One can imagine that there is a population of workers and a distribution of discount rates over this population. All workers, i, that have discount rates greater than the interest rate, i.e., $\delta_i > r$, will spend the first part of life at the low paying job and the second part at the high paying job; all workers, i, characterized by $\delta_i < r$ will spend the first part of the life at the high paying job and the second part at the low paying job. Hence, heterogeneity along the worker discount rate dimension can generate flows of workers moving from low to high paying jobs and at the same time flows of workers moving from high to low paying jobs.

4 Wages and Amenities are "Perfect Substitutes"

The model developed so far assumes that wages and amenities are not perfectly substitutable and preferences are quasi-linear in c and A. To see if these assumptions are the driving force behind job changes and/or the sequencing of job choice over a worker's lifetime, this section analyzes the case where wages and amenities are "perfect substitutes." Now, unlike the case when wages and amenities are not perfectly substitutable, the optimal consumption path of the private good, c, will depend critically on the path of amenity consumption. It will be shown that even when wages and amenities are perfect substitutes, workers may decide to change jobs and if the worker chooses to change jobs, the sequence of job choices is determined by the sign of $r - \delta$.

The momentary utility function is now assumed to take the form

$$u(c(t), A_i) = \ln(c(t) + A_i), \quad i = 1, 2. \tag{31}$$

For what follows it will be useful to define $w_i + A_i$ as the "aggregate wage" for job i and $c(t) + A_i$ as "aggregate consumption."

If the worker chooses job i at date $t = 0$ and remains there for the rest of his life, then the lifetime consumption-saving decision is determined by the solution to the following maximization problem,

$$\max_{\{c(t)\}} \int_0^1 e^{-\delta t} \ln(c_i(t) + A_i) dt \tag{32}$$

subject to

$$\dot{a}_i(t) = r a_i(t) + w_i - c_i(t) \tag{33}$$

$$a_i(0) = a_i(1) = 0 \tag{34}$$

$$c_i(t) \geq 0 \tag{35}$$

Qualitatively speaking, the only difference between this maximization problem and the one studied in the previous section is to be found in constraint (35). This constraint says that private consumption cannot be negative.[7] When this constraint binds it means the worker would, in some time periods, prefer to consume less than A_i, saving now to increase consumption in some other periods. Such a strategy, however, is not feasible because the amenity cannot be saved—it must be consumed in its entirety at each point in time.

The following propositions characterize the worker's optimal consumption profile and job choices when private consumption and amenities are perfect substitutes.

Proposition 2 *When constraint (35) does not bind for either job, the worker will choose that job which has the highest aggregate wage, say job i, and his aggregate consumption profile is given by*

$$c_i(t) + A_i = (w_i + A_i) \frac{\int_0^1 e^{-rt} dt}{\int_0^1 e^{-\delta t} dt} e^{(r-\delta)t}, \quad t \in [0,1]. \tag{36}$$

If the aggregate wages for the jobs are the same, then at any point in time the worker is indifferent between choosing job 1 or job 2.

Proof. See appendix. ■

Since the private good and the amenity are perfect substitutes and aggregate consumption is always greater than A_2—since constraint (35) does not bind—the worker will choose that job which has the highest lifetime aggregate income or, equivalently, the highest aggregate wage, $w_i + A_i$.

[7] Such a constraint was not required in the previous section's model because the marginal utility of private consumption is infinite when private consumption is zero.

As in the case where the private good and the amenity were not perfect substitutes, we find that the consumption profile is determined by the sign of $r-\delta$: if $r>\delta$, the consumption profile is increasing, if $r<\delta$, it is decreasing and if $r=\delta$, then it is flat. Note that when $r=\delta$, constraint (35) can never bind because if the worker chooses job i, the optimal (aggregate) consumption is given by w_i+A_i for all $t\in[0,1]$.

Proposition 3 *Suppose that constraint (35) binds for* $i=2$ *and possibly for* $i=1$. *(i) If* $w_1+A_1\geq w_2+A_2$, *then the worker will spend his entire life at job 1. (ii) If* $w_1+A_1<w_2+A_2$ *and the difference between the instantaneous utilities is sufficiently large, then worker will spend his entire life at job 2. (iii) If* $w_1+A_1<w_2+A_2$ *and the difference between instantaneous utilities is not too large, then if* $r>\delta$, *the worker will select job 1 at* $t=0$ *and will move to job 2 at time* $t=t_{12}^*>0$; *if* $r<\delta$, *he will choose job 2 at* $t=0$ *and move to job 1 at* $t=t_{21}^*>0$. *(iv) The worker's consumption profile is weakly increasing when* $r>\delta$ *and weakly decreasing when* $r<\delta$.

Proof. See appendix. ■

When $w_1+A_1\geq w_2+A_2$, job 1 "dominates" job 2 in two dimensions. First, job 1 has a higher aggregate lifetime income. Second, since $A_1<A_2$, job 2 will result in a more constrained aggregate consumption profile, compared to job 1, since aggregate consumption is unable to fall below A_2 for job 2 but is able to for job 1. Put another way, even when $w_1+A_1=w_2+A_2$, job 1 can always replicate any feasible aggregate consumption profile from job 2, but not vice-versa. When $w_1+A_1<w_2+A_2$ and the difference between aggregate wages is sufficiently large, although the aggregate consumption profile for job 2 is still constrained relative to job 1, job 2 offers a sufficiently higher lifetime aggregate income to more than compensate for the constrained aggregate consumption profile. As a result, the worker will choose job 2 for his entire life.

For both of these cases, the worker's optimal private consumption profile, as in the previous analysis, depends upon the relative magnitudes of r and δ. When $r>\delta$, the worker's optimal private consumption path is non-decreasing. The worker's private consumption associated with job i, $c_i(t)$, is given by

$$c_i(t)=\begin{cases}0 & \text{for } t\in[0,t_i^*]\\ A_i\left(e^{(r-\delta)(t-t_i^*)}-1\right) & \text{for } t\in(t_i^*,1],\end{cases} \tag{37}$$

where constraint (35) binds for $t \in [0, t_i^*]$ and does not for $t \in (t_i^*, 1]$; the worker consumes only the amenity when constraint (35) binds and when it does not, the worker's aggregate consumption grows at the rate of $r - \delta$. Since $r - \delta > 0$, which implies that the optimal private consumption profile is non-decreasing, the worker's private consumption will be constrained during the first part of his life. When $r < \delta$, the worker's optimal private consumption path is non-increasing. The worker's optimal private consumption associated with job i, $c_i(t)$, is given by

$$c_i(t) = \begin{cases} A_i \left(e^{(r-\delta)\left(t_i^* - t\right)} - 1 \right) & \text{for } t \in [0, t_i^*] \\ 0 & \text{for } t \in (t_i^*, 1], \end{cases} \tag{38}$$

where constraint (35) binds for $t \in [0, t_i^*]$ and does not for $t \in (t_i^*, 1]$. Since $r - \delta < 0$, which implies that the optimal private consumption profile is non-increasing, the worker's private consumption will be constrained in the latter part of his life.

Perhaps the most interesting case is when $w_1 + A_1 < w_2 + A_2$ *and* the difference between aggregate wages is "not too large." In this situation, by choosing job 1 (instead of job 2) over some part of his life, the worker obtains a less constrained aggregate consumption profile and by choosing job 2 (instead of job 1) over the remainder he is able to generate a higher a lifetime income. To see this, and without loss of generality, assume that $r > \delta$. In Figure 1, c_1^* represents the aggregate consumption profile if the worker stays forever in job 1. Suppose that a worker chooses job 1 and follows the consumption profile c_1^* until time $\hat{t}$, where $c_1^*(\hat{t}) = A_2$, at which time switches to job 2 for the remainder of life. Since $w_2 + A_2 > w_1 + A_1$, the worker will be able to consume more than $c_1^*(t)$ in all $t \geq \hat{t}$; hence it is optimal for the worker to change jobs. The *optimal* aggregate consumption profile for the individual, c_{12}^*, and the optimal time to change jobs, date t_{12}^*, are described in Figure 1.

When $r > \delta$ and constraint (35) binds, the worker consumes A_1 for $t \in [0, t_1^*]$, after which aggregate consumption grows at the rate of $r - \delta$. The critical dates, t_1^* and t_{12}^*, are determined by the equations:

$$c_{12}^*(t_{12}^*) = A_1 e^{(r-\delta)\left(t_{12}^* - t_1^*\right)} = A_2 \tag{39}$$

and

$$A_1(\int_0^{t_1^*} e^{-rt}dt + \int_{t_1^*}^1 e^{(r-\delta)(t-t_1^*)}dt) = (w_1+A_1)\int_0^{t_{12}^*} e^{-rt}dt + (w_2+A_2)\int_{t_{12}^*}^1 e^{-rt}dt. \tag{40}$$

The first equation says that the optimal aggregate consumption at the time the worker changes jobs, date t_{12}^*, is equal to A_2. The second equation simply says that lifetime aggregate consumption equals lifetime aggregate wages.

Hence, when constraint (35) binds, $w_2+A_2 > w_1+A_1$ and the difference between aggregate wages is not "too big," then, if $r > \delta$, the worker will initially choose job 1 and will ultimately switch to job 2. In this case the worker will be observed to move from a high paying job, w_1, to a low paying job, w_2. If $r < \delta$, the worker's optimal aggregate consumption profile will be non-increasing and he will initially choose job 2 and will ultimately switch to job 1, when aggregate consumption equals A_2. In this case, the worker will be observed moving from a low paying job, w_2, to a high paying one, w_1. So just as in the case where the wage and amenity are not substitutable, a model where wages and amenities are perfectly substitutable can have workers moving from high to low or from low to high paying jobs. And, if workers do change jobs, the direction of the movement depends on the relative magnitude of the rate of time preference.

5 Data Regarding Job Changes

It was mentioned in the introduction that the model makes predictions on wage changes over the life cycle. More specifically, it was shown that depending on the relationship between the interest rate and the rate of time preference, individuals may choose to move from higher to lower paying jobs. Therefore, it would be useful to know the extent to which workers move from higher to lower wage jobs.

Between 1984 and 1992 the Panel Study of Income Dynamics (PSID) asked individuals about their current and previous employer. For those workers who changed employers, a number of questions were asked: Their reasons for leaving the last employer; their wage with the last and current employer; when they left their last employer and when they began their current employment. The actual question for 1989 in the PSID and choice of response was:[8]

[8]The question and responses are slightly different for some years.

Question:

What happened with that employer—did the company go out of business, were you (HEAD) laid off, did you quit, or what?

Responses:

1. Company folded/changed hands/moved out of town; employer died/went out of business 1989
2. Strike; lockout
3. Laid off; fired
4. Quit; resigned; retired; pregnant; needed more money; just wanted a change in jobs; was self- employed before
5. Other; transfer; any mention of armed services
6. Job was completed; seasonal work; was a temporary job
7. NA; DK
8. Inap.: not working for money now; no other main-job employer during 1988; still working for other employer

After responding that an employer change took place, some follow-up questions were asked. In particular the worker was asked about their wage when the job ended with their previous employer and how much they earned when they started with their new employer. In addition, the date of the ending of the last job and beginning of the current job was also asked. Reported wages were converted to real wages using the monthly CPI since the dates of job endings and beginnings are given as a month within the year.

For the nine years of data (1984-1992) containing the above question, there are 42,765 observations where the respondent had positive income, was either head of the household or spouse of the head, and between the ages 18 and 70. The numbers in Table 1 and Table 2 are averages using all employer changes throughout all of the years. That is, each job change is considered one observation and no account is taken of the fact that some individuals in the data change employers several times while others may change only once.

From that population there were 3,599 people who changed employers for any reason. Table 1 shows summary statistics for all employer changers in the PSID from 1984-1992.

Table 1: All Job Changers

	To Lower Wage		To Same Wage		To Higher Wage	
	mean	std.	mean	std.	mean	std.
% of Job Changers	0.421	0.494	0.084	0.277	0.495	0.500
Age	33.6	9.62	34.5	10.2	32.6	9.06
Months Between Jobs	1.49	2.03	0.003	.057	0.906	1.51
N	3,599					

However, as mentioned above, this paper is concerned with those workers who answered with response #4. Table 2 provides summary statistics for those who changed employers voluntarily. There were 2,313 observations of employer changes between 1984 and 1992.

Table 2: Voluntary Job Changers

	To Lower Wage		To Same Wage		To Higher Wage	
	mean	std.	mean	std.	mean	std.
% of Job Changers	0.424	0.494	0.048	0.214	0.528	0.499
Age	32.7	9.13	33.4	10.8	32.0	8.58
Months Between Jobs	1.32	1.88	0.009	.095	0.920	1.50
N	2,313					

Though the majority of voluntary job changers, 53%, move to higher paying jobs, a very large proportion of voluntary job changers, 42.5%, move to jobs that pay lower wages. There is very little difference in age between those moving to higher or lower paying jobs, around 32-33 years of age. The median percentage change in real wages for those moving to lower paying jobs is -17.8%, while the median for those moving to higher paying jobs is nearly 20%, as can be seen in table 3. The challenge posed by this data is to provide a coherent theory of why a large fraction of workers move to higher paying jobs, while at the same time, a significant fraction of workers move to lower paying jobs.

Table 3: Wage Changes (%) for Voluntary Job Changers

	Quantiles				
	10%	25%	50%	75%	90%
moved to:					
lower wages	-2.03	-7.46	-17.8	-40.5	-72.5
higher wages	4.08	9.43	19.8	41.4	73.6

Postel-Vinay and Robin (2002) and Connolly and Gottschalk (2002) motivate a worker moving to a lower paying job by the potential for higher wage growth, compared to the worker's current job. Although it is possible to track wage growth before and after the switch, the PSID only asked these job change questions between 1984 and 1992, so that there are not many years before or after the job change. In any event, it is possible to look at those who voluntarily changed jobs to lower paying jobs exactly in the middle year of the data, 1988, and examine their wage growth four years before and four years after the job change. The results show very little difference between wage growth before and after the employer change. In the four years before 1988 average annualized real wage growth was 6.84% while the for the four years after it was 5.32%. However, due to the small sample size (39 individuals) the standard deviations are quite large, 23.9 and 19.3 respectively.

6 Conclusions

Individuals may rationally choose to move from high paying jobs to lower paying ones as part of an optimal lifetime plan. A key insight to this observation is that a job is more than just a wage; workers also care about non-wage dimensions of a job. In the data, the majority of workers who voluntarily change jobs, move to higher wages. Our model would identify these individual as having “relatively high” discount rates. The data also document that a large proportion of voluntary job changers move to lower paying jobs. Our model would identify these workers as patient, “relatively low” discount rate individuals.

The particular specification of preferences analyzed above are not the only ones that imply that the worker will want to change jobs. Computational experiments using a general CES utility function show that as long as the difference in instantaneous utility associated with each job is not too big, then worker’s will want to change jobs. So, just as the case when the wage and amenity are not perfectly substitutable, with CES preferences workers want to change jobs so that they can consume the “average” of both jobs bundles. The direction of job movement—from low to high or high to low paying jobs—is determined by the relative magnitude of the worker’s discount rate.

References

CONNOLLY, H. AND P. GOTTSCHALK, "Job Search with Heterogeneous Wage Growth–Transitions to "Better" and "Worse" Jobs," (2002), working Paper, Boston College.

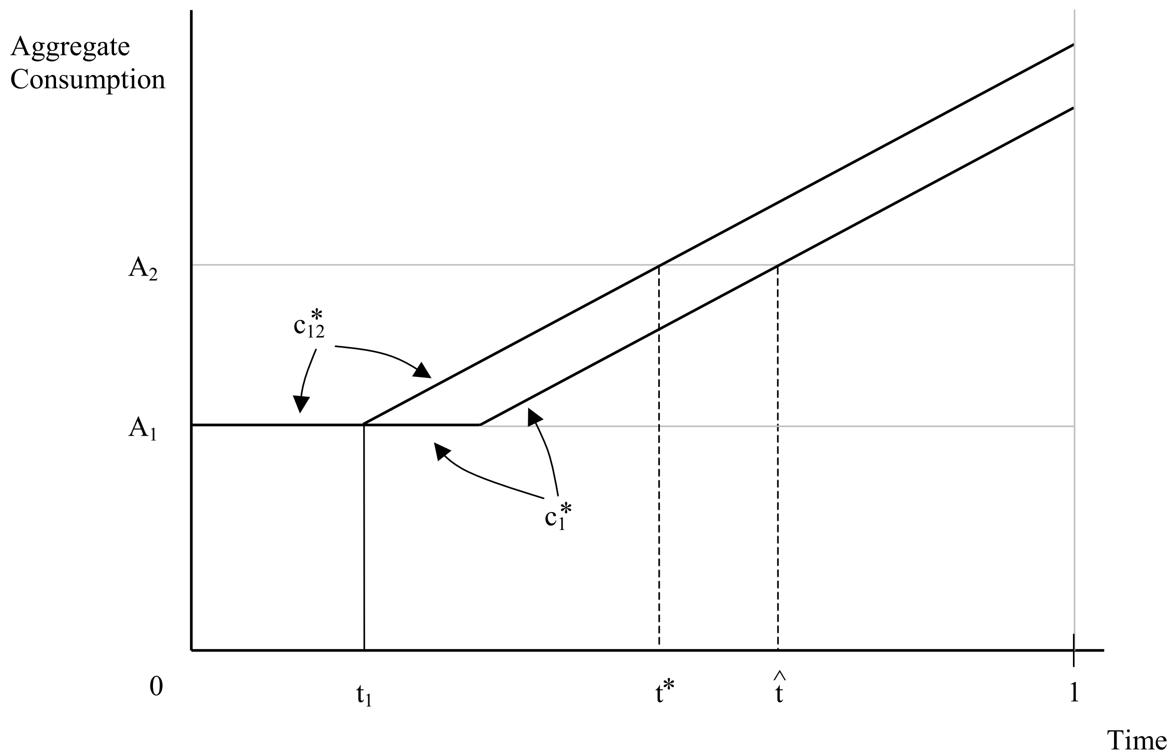
HWANG, H.-S., D. T. MORTENSEN, AND W. R. REED, "Hedonic Wages and Labor Market Search," *Journal of Labor Economics* 16 (1998), 815–847.

MORTENSEN, D., "Search Equilibrium and Real Business Cycles," (1990), working Paper, Northwestern University.

POSTEL-VINAY, F. AND J.-M. ROBIN, "Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica* 70 (2002), 2295–350.

ROSEN, S., "Learning and Experience in the Labor Market," *Journal of Human Resources* 7 (1972), 326–342.

Figure 1: Optimal Consumption Profile and Job Change when $r > \delta$



7 Appendix

Proof of Propostion 2. The current value Hamiltonian for maximization problem (32)-(34) is,

$$\mathscr{H} = \ln(c(t) + A_i) + \lambda(t)(a(t)r - w_i - c(t)). \tag{41}$$

The solution to this problem is given by,

$$\mathscr{H}_{c(t)} = \frac{1}{c(t) + A_i} - \lambda(t) = 0, \tag{42}$$

$$\dot{\lambda}(t) = -\mathscr{H}_{a(t)} + \lambda(t)\delta = -\lambda(t)(r - \delta). \tag{43}$$

Equations (42) and (43) imply that aggregate consumption, $c(t) + A_i$, grows at the rate of $r - \delta$, i.e.,

$$\frac{\frac{d(c(t)+A_i)}{dt}}{c(t) + A_i} = r - \delta. \tag{44}$$

Hence, when $r > \delta$, the worker's aggregate consumption is increasing over time; when $r < \delta$, it is decreasing; and when $r = \delta$, it is constant. Equation (44) can be rewritten as

$$c(t) + A_i = (c(0) + A_i)\, e^{(r-\delta)t} dt. \tag{45}$$

Using (45), the worker's lifetime budget constraint,

$$\int (c(t) + A_i)\, e^{-rt} dt = (w_i + A_i) \int e^{-rt} dt,$$

can be rewritten as

$$c(0) + A_i = (w_i + A_i)\, \frac{\int_0^1 e^{-rt}}{\int_0^1 e^{-\delta t}}.$$

Substituting this into (45), we get,

$$c(t) + A_i = (w_i + A_i) \frac{\int_0^1 e^{-rt} dt}{\int_0^1 e^{-\delta t} dt} e^{(r-\delta)t}, \quad t \in [0, 1],$$

which is equation (36) in the statement of proposition 2. If $w_i + A_i > w_j + A_j$, $i, j = \{1, 2\}, i \neq j$, then, from (36) $c_i(t) + A_i > c_j(t) + A_j$ for all t and the worker will remain at job i his entire life. If $w_i + A_i = w_j + A_j$, $i, j = \{1, 2\}, i \neq j$, then, from (36) $c_i(t) + A_i = c_j(t) + A_j$ for all t and the worker is indifferent between choosing job 1 and job 2. ■

Proof of Proposition 3. (i) If $w_1 + A_1 \geq w_2 + A_2$, then the worker will choose job 1 for his entire life: Job 1 offers a higher aggregate lifetime income and, since $A_2 > A_1$, job 1 offers consumption possibilities that are not obtainable under job 2, e.g., it is possible for aggregate consumption to be less than A_2 at job 1 but not at job 2. The current value Hamiltonian for this problem is

$$\mathscr{H}_1 = \ln(c(t) + A_1) + \lambda(t)(a(t)r + w_1 - c(t)),$$
$$\text{s.t. } c(t) \geq 0.$$

The Lagrangian for this problem is

$$\mathscr{L}_1 = \ln(c(t) + A_1) + \lambda(t)(a(t)r + w_1 - c(t)) + \gamma(t)c(t)$$
$$\gamma(t) \geq 0, c(t) \geq 0, c(t)\gamma(t) = 0$$

The necessary conditions for optimal behavior are

$$\frac{\partial \mathscr{L}_1}{\partial c(t)} = \frac{1}{c(t) + A_1} - \lambda(t) + \gamma(t) = 0 \tag{46}$$

and

$$\dot{\lambda}(t) = -\frac{\partial \mathscr{H}_1}{\partial a(t)} + \lambda(t)\delta = -\lambda(t)(r - \delta).$$

This latter equality implies that

$$\lambda(t) = k_0 e^{-(r-\delta)t}, \tag{47}$$

where k_0 is a constant, to be determined. Suppose that $r > \delta$. We conjecture that the solution will take the form

$$c(t) + A_1 = \begin{cases} A_1, & 0 \leq t \leq t_1^* \\ A_1 e^{(r-\delta)(t - t_1^*)}, & t_1^* < t \leq 1 \end{cases} \tag{48}$$

To see this note that from (46), when $c(t) = 0$,

$$\gamma(t) = k_0 e^{-(r-\delta)t} - \frac{1}{A_1}. \tag{49}$$

Define t_1^* by

$$0 = k_0 e^{-(r-\delta)t_1^*} - \frac{1}{A_1}$$

or

$$k_0 = \frac{e^{(r-\delta)t_1^*}}{A_1}, \tag{50}$$

Since $\gamma(t)$ is a decreasing function of t (see (49))and $\gamma(t)\,c(t) = 0$, it must be the case that $\gamma(t) > 0$ for $t \in [0, t_1^*]$, $t_1^* > 0$; otherwise $\gamma(t) = 0$. When $\gamma(t) = 0$,

$$\frac{d\,(c\,(t) + A_1)\,/dt}{(c\,(t) + A_1)} = (r - \delta)\,, \tag{51}$$

and, since $c\,(t) = 0$ for $t \in [0, t_1^*]$,

$$c\,(t) + A_1 = A_1 e^{(r-\delta)\left(t - t_1^*\right)}. \tag{52}$$

All this implies that the worker's consumption profile is given by (48). From (52), the worker's budget constraint can be written as

$$w_1 \int_0^1 e^{-rt} dt = A_1 \int_{t^*}^1 \left(e^{(r-\delta)\left(t-t_1^*\right)} - 1\right) e^{-rt} dt;$$

this equation determines the value of t_1^*; given t_1^*, k_0 can be inferred from equation (50). The value of being in job 1 for the worker's entire life, V_1, is

$$V_1 = \int_0^{t_1^*} \ln(A_1)\, e^{-\delta t} dt + \int_{t_1^*}^1 \ln\left(A_1 e^{(r-\delta)\left(t-t_1^*\right)}\right) e^{-\delta t} dt.$$

When $r < \delta$, we conjecture that consumption will be characterized by

$$c\,(t) + A_1 = \begin{cases} A_1 e^{(r-\delta)\left(t_1^* - t\right)}, & 0 \le t \le t_1^* \\ A_1, & t_1^* < t \le 1 \end{cases}\,, \tag{53}$$

i.e., consumption is non-increasing. (Note: the value of t_1^* here is not the same value of t_1^* that was characterized for the case where $r > \delta$, above.) If $\gamma(t) > 0$, then, from (49), $\gamma(t)$ is a strictly increasing function of t. (The relationship between t_1^* and k_0 is still defined by (50).) Hence, it must be the case that $\gamma(t) > 0$ for $t \in [t_1^*, 1]$, $t_1^* > 0$; otherwise $\gamma(t) = 0$. When $c\,(t) > 0$, (i.e., $\gamma(t) = 0$) consumption growth is characterized by (51), and since $c\,(t) = 0$ for $t \in [t_1^*, 1]$,

$$c\,(t) + A_1 = A_1 e^{(r-\delta)\left(t_1^* - t\right)}. \tag{54}$$

From (54), the worker's budget constraint can be written as

$$w_1 \int_0^1 e^{-rt} dt = A \int_0^{t_1^*} \left(e^{(r-\delta)\left(t_1^* - t\right)} - 1\right) e^{-rt} dt;$$

from this equation the value of t_1^* can be determined and given t_1^*, k_0 can be inferred from equation (50). The value of being in job 1, V_1, is given by

$$V_1 = \int_0^{t_1^*} \ln\left(A_1 e^{(r-\delta)\left(t_1^*-t\right)}\right) e^{-\delta t} + \int_{t_1^*}^1 \ln\left(A_1\right) e^{-\delta t} dt.$$

(ii) If $w_1 + A_1 < w_2 + A_2$ and the difference is sufficiently large, then the worker will stay at firm 2 for his entire life. (For example, if $w_2 = w_1 - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small and $A_2 \gg A_1$, then job 2 is obviously preferred to job 1. What precisely we mean by "sufficiently large" will be described in part (iii) of this proof.) The characterization of the optimal consumption profile follows almost exactly from what was described in part (i) above, except that a "2" is substituted everywhere there is a "1." So, if, for example, $r > \delta$ and the worker stays at firm 2 for his entire life, his optimal consumption profile is given by

$$c(t) + A_2 = \begin{cases} A_2, & 0 \le t \le t_2^* \\ A_1 e^{(r-\delta)\left(t-t_2^*\right)}, & t_2^* < t \le 1 \end{cases}. \tag{55}$$

and the value of this job to the worker, V_2, is

$$V_2 = \int_0^{t_2^*} \ln\left(A_2\right) e^{-\delta t} dt + \int_{t_2^*}^1 \ln\left(A_2\left(e^{(r-\delta)\left(t-t_2^*\right)} - 1\right)\right) e^{-\delta t} dt.$$

If, on the other hand, $r < \delta$ and the worker stays at firm 2 for his entire life, his optimal consumption profile is given by

$$c(t) + A_2 = \begin{cases} A_2 e^{(r-\delta)\left(t_2^*-t\right)}, & 0 \le t \le t_2^* \\ A_2, & t_2^* < t \le 1 \end{cases}, \tag{56}$$

and the lifetime utility associated with this consumption profile, V_2, is

$$V_2 = \int_0^{t_2^*} \ln\left(A_2 e^{(r-\delta)\left(t_1^*-t\right)}\right) e^{-\delta t} + \int_{t_2^*}^1 \ln\left(A_2\right) e^{-\delta t} dt.$$

(iii) If $w_1 + A_1 < w_2 + A_2$ and the difference is not "too large," then worker will choose to work at both jobs 1 and 2 over his lifetime and the sequencing of jobs depends on the relative magnitudes of r and δ. To see this, consider first the case where $w_1 + A_1 = w_2 + A_2$. Assuming that the constraint (35) binds for some t for $i = 2$, i.e., when the worker stays at job 2 for his entire life, and possibly binds for some t when $i = 1$, it is necessarily the case that $V_1 > V_2$. Since $A_1 < A_2$, it is possible to replicate the job 2 consumption profile—either (55) or (56)—with job

1. But the consumption profiles for the jobs differ (i.e., compare (48) and (53) with (55) and (56), respectively), which necessarily implies that job 1 provides a higher lifetime utility than job 2. Suppose now that $w_1 + A_1 = w_2 + A_2 - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. Since $V_1(\varepsilon = 0) > V_2(\varepsilon = 0)$, by continuity, $V_1(\varepsilon) > V_2(\varepsilon)$ when $\varepsilon > 0$ and arbitrarily small. In fact, when $\varepsilon > 0$, the worker is able to obtain a higher lifetime utility than V_1. To see this, suppose that when $c_1(t) + A_1 \geq A_2$, where $c_1(t) + A_1$ is given by either (48) or (53), the worker switches from job 1 to job 2. Since $w_1 + A_1 < w_2 + A_2$, the worker will be able to obtain higher levels of consumption than $c_1(t) + A_1$ when at job 2. Hence, if $\varepsilon > 0$ is small, the worker will have an incentive to work at both jobs over his lifetime rather than spend his entire life at job 1. For example, if $r > \delta$, the worker's consumption will be weakly increasing and he will start off with job 1; his consumption will be A_1 for $t \in [0, t_1^*]$ and he will remain at job 1 until time $t_{12}^* > t_1^*$, after which he switches to job 2 and remains there for the rest of his life. The worker's optimal aggregate consumption profile is given by

$$c(t) + A_1 = \begin{cases} A_1, & 0 \leq t \leq t_1^* \\ A_1 e^{(r-\delta)\left(t - t_1^*\right)}, & t_1^* < t \leq 1 \end{cases}. \tag{57}$$

(If the constraint $c(t) \geq 0$ never binds for job 1, then $t_1^* = 0$.) The worker will change jobs at time t_{12}^* when $c(t) + A_1 = A_2$ or when

$$A_1 e^{(r-\delta)\left(t_{12}^* - t_1^*\right)} = A_2 \tag{58}$$

From the worker's lifetime aggregate budget constraint we have

$$A_1 \int_0^{t_1^*} e^{-rt} dt + A_1 \int_{t_1^*}^1 e^{(r-\delta)\left(t - t_1^*\right)} e^{-rt} dt = (w_1 + A_1) \int_0^{t_{12}^*} e^{-rt} dt + (w_1 + A_1) \int_{t_{12}^*}^1 e^{-rt} dt \tag{59}$$

Equations (58) and (59) determine t_1^* and t_{12}^*. The lifetime utility associated with consumption profile (57), V_{12}, is

$$V_{12} = \ln(A_1) \int_0^{t_1^*} e^{-\delta t} dt + \int_{t_1^*}^1 \ln\left(A_1 e^{(r-\delta)\left(t - t_1^*\right)}\right) e^{-\delta t} dt.$$

If $r > \delta$, then the worker's consumption will be weakly decreasing and he will start off with job 2. His consumption will be strictly decreasing for $t \in [0, t_2^*]$ and will be equal to A_1 for $t \in [t_2^*, 1]$; he will remain at job 2 until time $t_{21}^* < t_2^*$, after which he switches to job 1 and remains there for the rest of his life. The worker's optimal consumption profile is given by

$$c(t)+A_1=\begin{cases} A_1e^{(r-\delta)\left(t_2^*-t\right)}, & 0\le t\le t_2^* \\ A_1, & t_2^*<t\le 1 \end{cases}, \tag{60}$$

The worker will change jobs at time t_{21}^* when $c(t)+A_1=A_2$ or when

$$A_1e^{(r-\delta)\left(t_2^*-t_{21}^*\right)}=A_2 \tag{61}$$

From the worker's lifetime aggregate budget constraint we have

$$A_1\int_0^{t_2^*}e^{(r-\delta)\left(t_2^*-t\right)}e^{-rt}dt+A_1\int_{t_2^*}^1 e^{-rt}dt=(w_2+A_2)\int_0^{t_{21}^*}e^{-rt}dt+(w_1+A_1)\int_{t_{21}^*}^1 e^{-rt}dt \tag{62}$$

Equations (61) and (62) determine t_2^* and t_{21}^*. The lifetime utility associated with consumption profile (60), V_{21}, is

$$V_{12}=\int_0^{t_2^*}\ln\left(A_1e^{(r-\delta)\left(t_2^*-t\right)}\right)e^{-\delta t}dt+\ln(A_1)\int_{t_2^*}^1 e^{-\delta t}dt.$$

We will say that the difference between the aggregate wages is "not too large" when $\max\left\{V_2,V_{ij}\right\}=V_{ij}$—where $i=1$ and $j=2$ if $r>\delta$ and $i=2$ and $j=1$ if $r<\delta$—and is "sufficiently large" when $\max\left\{V_2,V_{ij}\right\}=V_2$. (This is motivated by the fact, that holding w_1+A_1 constant and increasing w_2+A_2, by either increasing w_2 or A_2, the difference V_2-V_{ij} will increase as w_2+A_2 increases. Hence, if the difference between aggregate wages is not too large, the worker will spend parts of his life in both jobs. If $r>\delta$, he will start off in job 1 and will move to job 2; if $r<\delta$, then he will start off in job 2 and will move to job 1. If the difference between aggregate wages is sufficiently large, then the worker will remain in job 2 his entire life.

(iv) If $c(t)>A_i$ when the worker is at job i, then his consumption grows at the rate $r-\delta$. Therefore, if $r>\delta$, then the worker's consumption profile will be weakly increasing and if $r<\delta$, then his consumption profile will be weakly decreasing. ■